Warm-UP

Find the vertex and all intercepts by completing the square then graph.

$f(x) = \left[-\frac{1}{2}x^2 + 3x\right] - 2$ ignore for now

$= -\frac{1}{2}\left[\frac{-\frac{1}{2}x^2}{-\frac{1}{2}} + \frac{3x}{-\frac{1}{2}}\right] - 2$

$3 \div -\frac{1}{2}$
$= 3x - \frac{2}{1}$
$= -6$

$= -\frac{1}{2}\left[(x^2 - 6x + k) - k\right] - 2$

$k = \left(\frac{1}{2}\text{middle}\right)^2$

$= \left(\frac{1}{2}(-6)\right)^2 = (-3)^2 = 9$

$= -\frac{1}{2}\left[(x^2 - 6x + 9) - 9\right] - 2$

$= -\frac{1}{2}\left[(x-3)^2 - 9\right] - 2$

$= -\frac{1}{2}(x-3)^2 + \left(-\frac{1}{2}\right)(-9) - 2$

$= -\frac{1}{2}(x-3)^2 + \frac{9}{2} - \frac{2}{1}$ $\times 2$ $= -\frac{1}{2}(x-3)^2 + \frac{5}{2}$

2.5

2.5

V: $(3, \frac{5}{2})$

y-int: $f(0) = -\frac{1}{2}(0-3)^2 + \frac{5}{2} = -\frac{9}{2} + \frac{5}{2}$

$= -\frac{4}{2} = -2 \Rightarrow \underline{(0, -2)}$

x int: $-\frac{1}{2}(x-3)^2 + \frac{5}{2} = 0$

$-2x \quad -\frac{1}{2}(x-3)^2 = -\frac{5}{2} \quad x-2$

$(x-3)^2 = 5$

$x-3 = \pm\sqrt{5}$

$x = 3 \pm \sqrt{5}$

$x_1 (3+\sqrt{5}, 0) \quad x_2 (3-\sqrt{5}, 0)$ or $(3 \pm \sqrt{5}, 0)$

For graph $(5.2, 0), (0.76, 0)$

Ex 1

A rancher is setting up a fenced rectangular area for her sheep next to a cliff (no fencing needed next to the cliff). If she has 1000m of fencing in total to use and wants to enclose the largest area:

a) What dimensions should the rectangle be?

b) What is the total enclosed area?

$A = xy = x(1000 - 2x) \Rightarrow A = -2x^2 + 1000x$

$1000 = 2x + y$

$1000 - 2x = y$

$a = -2 \quad b = 1000 \quad c = 0$

$h = -\frac{b}{2a} = \frac{-1000}{2(-2)} = \frac{-1000}{-4} = 250$

x value

$k = c - \frac{b^2}{4a} = 0 - \frac{(1000)^2}{4(-2)} = 125\,000$

A value

$V: (250, 125\,000)$

$y = 1000 - 2(250) = 500$ a) 250m × 500m b) $A = (250)(500) = 125\,000$

Ex 2

A hotel is trying to decide on a nightly rate to maximize income. When they charge $80 a night, 75% of the 400 rooms are booked. A survey of customers shows that increasing the price by $5 would decrease the number of customers by 10. Find the nightly rate and number of customers that maximizes their income.

Income = # of rooms occupied • cost per room $= (300 - 10x)(80 + 5x)$

400×0.75

x = the number of times we increase the price

$I = (300 - 10x)(80 + 5x) = 24\,000 - 800x + 1500x - 50x^2$

$= 24000 + 700x - 50x^2$

$a = -50 \quad b = 700 \quad c = 24\,000$

$h = -\frac{b}{2a} = \frac{-(700)}{2(-50)} = +7$

# of times we should increase price

a) $80 + 5(7) = 80 + 35 = \$115$

b) $300 - 10(7) = 300 - 70 = 230$

c) When does the apple reach its maximum height?
d) When does it hit the ground?

a) $t=0 \Rightarrow H(0) = -5(0)^2 + 12(0) + 11 = 11\text{ m}$

b) vertex: $a=-5$, $b=12$, $c=11$

$h = \frac{-b}{2a} = \frac{-12}{2(-5)} = \frac{12}{10} = \frac{6}{5} = 1.2$

How long to get to max height

$k = f\left(\frac{6}{5}\right) = -5\left(\frac{6}{5}\right)^2 + 12\left(\frac{6}{5}\right) + 11$

$= -5\left(\frac{36}{25}\right) + \frac{72}{5} + 11$

$= -\frac{36}{5} + \frac{72}{5} + \frac{55}{5} = \frac{91}{5} = 18.2\text{ m}$

c) $t = 1.2\text{s}$

d) $H = 0 = -5t^2 + 12t + 11$

$0 = -5\left(t - \frac{6}{5}\right)^2 + \frac{91}{5}$

$\div -5 \quad \frac{-91}{5} = -5\left(t - \frac{6}{5}\right)^2 \quad \div -5$

$\frac{91}{25} = \left(t - \frac{6}{5}\right)^2 \Rightarrow \pm\sqrt{\frac{91}{25}} = t - \frac{6}{5} \Rightarrow t = \frac{6}{5} \pm \sqrt{\frac{91}{25}}$

$t = \frac{6}{5} + \sqrt{\frac{91}{25}} = 3.11\text{s}$

$t = \frac{6}{5} - \sqrt{\frac{91}{25}} = -0.71\text{s}$