Vertex Short Cut (2.3)

(2.1/2.2 Quiz next class)

Warm-Up

① Find the vertex by completing the square

$$f(x) = [-x^2 + 6x] - 27$$

$$= -1(x^2 - 6x) - 27$$

$$= -1[(x^2 - 6x + k) - k] - 27$$

$$\left(\frac{\text{middle}}{2}\right)^2$$

$$= \left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$$

$$= -1[(x^2 - 6x + 9) - 9] - 27$$

$$= -1[(x - 3)^2 - 9] - 27$$

$$= -(x - 3)^2 + 9 - 27$$

$$= -(x - 3)^2 - 18$$

$h = 3$, $k = -18$

$$\boxed{(3, -18)}$$

② Find the x-intercepts by completing the square

$$f(x) = (4x^2 + 7x) + \frac{3}{2}$$

$$= 4\left(\left(x^2 + \frac{7}{4}x + k\right) - k\right) + \frac{3}{2}$$

$$\left(\frac{\text{Middle}}{2}\right)^2 = \left(\frac{7}{4} \div \frac{2}{1}\right)^2$$

$$= \left(\frac{7}{4} \times \frac{1}{2}\right)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

$$= 4\left[\left(x^2 + \frac{7}{4}x + \frac{49}{64}\right) - \frac{49}{64}\right] + \frac{3}{2}$$

$$= 4\left[\left(x + \frac{7}{8}\right)^2 - \frac{49}{64}\right] + \frac{3}{2}$$

$$= 4\left(x + \frac{7}{8}\right)^2 - \frac{49}{16} + \frac{3}{2}$$

$$= 4\left(x + \frac{7}{8}\right)^2 - \frac{49}{16} + \frac{24}{16}$$

$$f(x) = 4\left(x + \frac{7}{8}\right)^2 - \frac{25}{16} = 0$$

$$\div 4 \quad 4\left(x + \frac{7}{8}\right)^2 = \frac{25}{16} \quad \div 4$$

$$\left(x + \frac{7}{8}\right)^2 = \frac{25}{64}$$

$$\left(x+\frac{7}{8}\right)^2=\frac{25}{64}$$

$$x+\frac{7}{8}=\pm\sqrt{\frac{25}{64}}$$

$$x+\frac{7}{8}=\pm\frac{5}{8}$$

$$x=-\frac{7}{8}\pm\frac{5}{8}$$

$$x_1=-\frac{7}{8}+\frac{5}{8}=-\frac{2}{8}=-\frac{1}{4}$$

$$x_2=-\frac{7}{8}-\frac{5}{8}=-\frac{12}{8}=-\frac{3}{2}$$

$$\left(-\frac{1}{4},0\right),\ \left(-\frac{3}{2},0\right)$$

Vertex Shortcut (2.3)

To get the shortcut equation, we need to Complete the Square for a general parabola.

$$f(x) = [ax^2 + bx] + c$$

$$= a\left[x^2 + \frac{b}{a}x\right] + c$$

$$\left(\frac{\text{middle}}{2}\right)^2 = \left(\frac{b}{a} \div \frac{2}{1}\right)^2$$

$$= \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$= a\left[\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a^2}\right] + c$$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right] + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

$$a(x-h)^2 + k$$

Vertex: (h, k)

$$\boxed{h = -\frac{b}{2a} \qquad k = -\frac{b^2}{4a} + c}$$

$a(x-h)^2+k$

Vertex: (h, k)

$$\boxed{h = -\frac{b}{2a} \qquad k = -\frac{b^2}{4a} + c}$$

Note: 'a' is the same for general and standard form

Ex 1 Find the vertex of $f(x) = -\frac{1}{4}x^2 + 2x - 3$

$ax^2 + bx + c$

$a = -\frac{1}{4}$

$b = 2$

$c = -3$

$$h = -\frac{b}{2a} = -\frac{(2)}{2(-\frac{1}{4})}$$

$$= -\frac{2}{-\frac{1}{2}} = -2 \times \left(-\frac{2}{1}\right)$$

$$\underline{h = 4}$$

← x value of vertex

① $k = -\frac{b^2}{4a} + c$

② $f(4) = -\frac{1}{4}(4)^2 + 2(4) - 3$

Vertex

①

$$k = -\frac{b^2}{4a} + c$$

$$= -\frac{(2)^2}{4(-\frac{1}{4})} + (-3)$$

$$= \frac{-4}{-1} + (-3)$$

$$k = 4 + (-3) = \underline{1}$$

②

$$f(4) = -\frac{1}{4}(4)^2 + 2(4) - 3$$

$$= -\frac{1}{4}(16) + 8 - 3$$

$$= -4 + 8 - 3$$

$$= 4 - 3 = \underline{1}$$

$\boxed{\underline{\text{Vertex}}: (4,1)}$

Ex 2

Find the vertex, and all intercepts (x and y) then graph $f(x) = -3x^2 + 6x + 4$

<u>vertex</u>: $h = \frac{-b}{2a} = \frac{-6}{2(-3)} = \frac{-6}{-6} = \underline{1}$

$$k = f(1) = -3(1)^2 + 6(1) + 4$$

$\boxed{(1,7)}$ $= -3 + 6 + 4 = \underline{7}$

y-int: $f(0) = \cancel{-3(0)^2} \cancel{+6(0)} + 4 = \underline{4}$

$$\boxed{(0,4)}$$

x-int: Use standard form

$a = -3$

$h = 1$

$k = 7$

$$f(x) = a(x-h)^2 + k$$

$$f(x) = -3(x-1)^2 + 7$$

(This is NOT completing the square)

$$0 = -3(x-1)^2 + 7$$

$$-7 = -3(x-1)^2$$

$$\frac{7}{3} = (x-1)^2$$

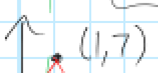
$$\pm\sqrt{\frac{7}{3}} = x - 1 \quad \Rightarrow \quad x = 1 \pm \sqrt{\frac{7}{3}}$$

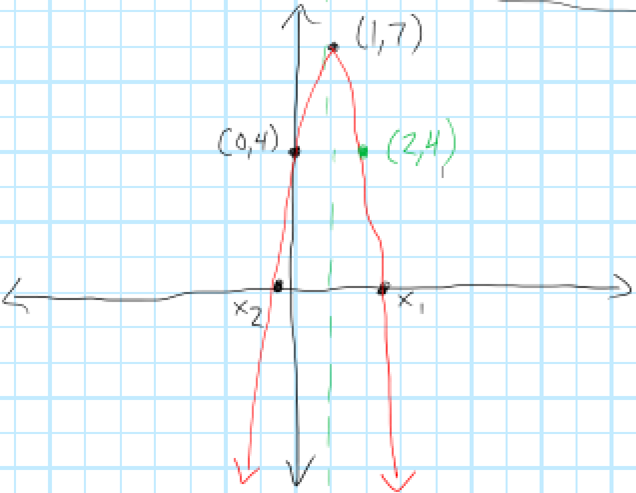
$$x_1 = 1 + \sqrt{\frac{7}{3}} \approx \boxed{2.53}$$

$$x_2 = 1 - \sqrt{\frac{7}{3}} \approx \boxed{-0.53}$$

For graphing

$$\boxed{\left(1+\sqrt{\frac{7}{3}}, 0\right), \left(1-\sqrt{\frac{7}{3}}, 0\right)}$$




(1,7)
(0,4)
(2,4)
x_2
x_1