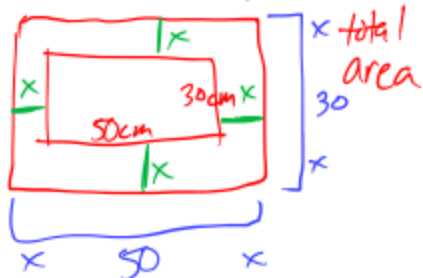


Ex 1 A 30cm by 50cm picture has a frame surrounding it. If the frame is the same width all around, and the total area of the frame and picture is 1836 cm², how wide is the frame?



$$1836 = (50+2x)(30+2x)$$

$$1836 = 1500 + 60x + 100x + 4x^2$$

$$0 = 4x^2 + 160x - 336$$

$$0 = 4 \left[(x^2 + 40x + k) - k \right] - 336$$

$$0 = 4 \left[(x^2 + 40x + 400) - 400 \right] - 336$$

↑ $(\frac{1}{2}40)^2 = (20)^2 = 400$

$$0 = 4(x+20)^2 - 400 \cdot 4 - 336 \Rightarrow 0 = 4(x+20)^2 - 1936$$

Check $x=2$

$$1836 = (50+4)(30+4)$$

$$1836 = 1836$$

✓

Ex 2 The sum of a number and three times its reciprocal is $\frac{28}{3}$. What is the number?

x - the number

$$x \left(x + 3 \cdot \frac{1}{x} \right) = \left(\frac{28}{3} \right) x$$

$$3(x^2 + 3) = \left(\frac{28}{3} x \right) \cdot 3$$

$$3x^2 + 9 = 28x$$

$$3x^2 - 28x + 9 = 0$$

$$\textcircled{\times} 27 \mid (-1, -27)$$

$$\textcircled{\oplus} -28 \mid -28$$

$$3x^2 - x - 27x + 9 = 0$$

$$x(3x-1) - 9(3x-1) = 0$$

$$(3x-1)(x-9) = 0$$

$$\begin{matrix} \uparrow \\ 3x-1=0 \\ \boxed{x = \frac{1}{3}} \end{matrix}$$

$$\begin{matrix} \uparrow \\ x-9=0 \\ \boxed{x = 9} \end{matrix}$$

Check $x=9$

$$9 + 3 \cdot \frac{1}{9}$$

$$9 + \frac{1}{3} = \frac{27}{3} + \frac{1}{3} = \frac{28}{3} \checkmark$$

$$1936 = 4(x+20)^2$$

$$\sqrt{484} = \sqrt{(x+20)^2}$$

$$\pm 22 = x + 20$$

$$x = -20 \pm 22$$

$$x = \textcircled{2}, \text{ } \cancel{-42} \leftarrow \text{neg width}$$

$x = \frac{1}{3}$

$$\frac{1}{3} + 3 \cdot \frac{1}{\left(\frac{1}{3}\right)} \leftarrow 1 \div \frac{1}{3} = 1 \cdot \frac{3}{1}$$

$$\frac{1}{3} + 3 \cdot 3 = \frac{1}{3} + 9 = \frac{28}{3} \checkmark$$

Ex 3 Mr. Raoul bikes to work, while Ms. Anderson drives to work, but they both live 8km away from the school. If they both leave at the same time, Ms. Anderson gets to school 12 minutes before Mr. Raoul. If Ms.

Ex 3 Mr. Raoul bikes to work, while Ms. Anderson drives to work, but they both live 8km away from the school. If they both leave at the same time, Ms. Anderson gets to school 12 minutes before Mr. Raoul. If Ms. Anderson drives 15km/h faster than Mr. Raoul can bike, what is the speed of each of them (answer to 1 decimal place)?

	Dist.	Speed	time
Mr. R	8km	r	t
Ms. A	8km	$r+15 \frac{\text{km}}{\text{h}}$	$t - 12 \text{ min}$ $t - 0.2 \text{ h}$

$D = S \cdot t$

$$8 = r \cdot t \rightarrow t = \frac{8}{r}$$

$$8 = (r+15)(t-0.2)$$

$$8 = (r+15)\left(\frac{8}{r} - 0.2\right)$$

$$8 = r \cdot \frac{8}{r} + 15 \cdot \frac{8}{r} + r \cdot (-0.2) + 15 \cdot (-0.2)$$

$$8 = 8 + \frac{120}{r} - 0.2r - 3$$

$$r \cdot 0 = \left(\frac{120}{r} - 0.2r - 3\right) \cdot r$$

$$0 = 120 - 0.2r^2 - 3r$$

$$r = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(-0.2)(120)}}{2(-0.2)} = \frac{3 \pm \sqrt{9 + 96}}{-0.4} = 18.1, -33.1$$

$$\begin{aligned} a &= -0.2 \\ b &= -3 \\ c &= 120 \end{aligned}$$

$r = \text{Speed}$

$$\text{Mr. R} \rightarrow 18.1 \frac{\text{km}}{\text{h}}$$

$$\text{Ms. A} \rightarrow 18.1 + 15 = 33.1 \frac{\text{km}}{\text{h}}$$

Ex 4 Mr. G and Mrs. Holman are marking PC 11 exams. Mrs. Holman can mark a class set of exams 10 minutes faster than Mr. G can. If they both work together, they can mark a class set in 1 hour. How long does it take each of them to mark a class set of exams (answer to the nearest minute)?

	Time to mark 1 class	Rate = $\frac{\text{Amount}}{\text{time}}$
Mr. G	$x \text{ min}$	$\frac{1 \text{ class}}{x \text{ min}} = \frac{1}{x}$
Mr. H	$x - 10 \text{ min}$	$\frac{1 \text{ class}}{x - 10 \text{ min}} = \frac{1}{x - 10}$

$$\text{Combined rate: } \frac{1}{x} + \frac{1}{x-10} = \frac{1 \text{ class}}{1 \text{ hour}} = \frac{1 \text{ class}}{60 \text{ min}}$$

$$x(x-10)\left(\frac{1}{x} + \frac{1}{x-10}\right) = \left(\frac{1}{60}\right)x(x-10)$$

$$\begin{aligned} &1 \text{ test/min} \\ &+ 2 \text{ test/min} \\ &\hline &3 \text{ test/min} \end{aligned}$$

$$60 \cdot (x-10 + x) = \left(\frac{1}{60} x(x-10) \right) \cdot 60$$

$$60x - 600 + 60x = x^2 - 10x$$

$$0 = x^2 - 10x - 60x - 600 + 600$$

$$0 = x^2 - 130x + 600$$

$$\begin{aligned} a &= 1 \\ b &= -130 \\ c &= 600 \end{aligned}$$

$$x = \frac{-(-130) \pm \sqrt{(-130)^2 - 4(1)(600)}}{2(1)}$$

$$= \frac{130 \pm \sqrt{16900 - 2400}}{2}$$

time to mark class in min \nearrow

$$x = 125.2, \cancel{48} \quad \leftarrow \text{rounded}$$

$$\text{Mr. G} \Rightarrow 125 \text{ min}$$

$$\begin{aligned} \text{Ms. H} &\Rightarrow 125 - 10 \\ &= 115 \text{ min} \end{aligned}$$