

TUESDAY, SEPTEMBER 16, 2013 3:24 PM

In general, we will be factoring quadratic trinomials of the form m^2 . $\underline{ax^2} + \underline{bx} + \underline{c}$

Today, we will be factoring

$$x^2 + bx + c \quad (a=1)$$

$$x^2 + bx + c = (x+q)(x+p)$$

\swarrow Factors

$$x^2 + bx + c = x^2 + qx + px + pq$$

$$x^2 + \underline{bx} + \underline{c} = x^2 + \underline{(q+p)x} + \underline{pq}$$

In order to factor $x^2 + bx + c$

We need to find p, q so that

$$p + q = b \quad (\text{AND}) \quad p \cdot q = c$$

NOTE: this ONLY works if

$$a = 1$$

$$\text{Ex 1} \quad \text{Factor } x^2 + 6x + 8$$

$$\begin{array}{r} (\times) +8 \\ \hline (\oplus) +6 \end{array} \left| \begin{array}{c} (2, 4), (1, 8), (-2, -4), (-1, -8) \\ 6 \end{array} \right. \quad \begin{array}{c} b \\ c \end{array}$$

$$x^2 + 6x + 8 = (x+2)(x+4)$$

Check $x^2 + 2x + 4x + 8$
 $x^2 + 6x + 8 \quad (1)$

$$\begin{array}{r} (\times) +8 \\ \hline (\oplus) +6 \end{array} \stackrel{(1)}{\Rightarrow} (2, 4) \stackrel{(2)}{\Rightarrow} (x+2)(x+4) \stackrel{(3)}{\Rightarrow}$$

$$\text{Ex 2} \quad \text{Factor } x^2 - 15 + 2x$$

$$\begin{array}{r} (\times) -15 \\ \hline (\oplus) +2 \end{array} \left| \begin{array}{c} (3, -5), (-3, 5), (1, -15), (-1, 15) \\ -2 \quad 2 \end{array} \right. \quad \begin{array}{c} b \\ c \end{array}$$

$$x^2 + 2x - 15 = (x+5)(x-3)$$

$$\text{Ex 3} \quad \text{Factor } x^2 - 5x - 4$$

$$\begin{array}{r} (\times) -4 \\ \hline (\oplus) -5 \end{array} \left| \begin{array}{c} (-2, 2), (1, -4), (-1, 4) \\ 0 \quad -3 \quad 3 \end{array} \right.$$

No factors exist ∴
Cannot be factored.

$$\begin{aligned}
 \underline{\text{Ex 4}} \quad & \text{Factor} \quad -2\cancel{x^3} + 2\cancel{x^2} + 24\cancel{x} \\
 & = x \left(\cancel{-2}x^2 + \cancel{2}x + \cancel{24} \right) \\
 & = -2x(x^2 - 1x - 12)
 \end{aligned}$$

$$\textcircled{\times} -12 | (1, -12), (-1, 12), (3, -4), (-3, 4), (6, -2), (2, -6)$$

$$\textcircled{+} -1 | -11 \quad 11 \quad -1$$

$$= \boxed{-2x(x+3)(x-4)}$$

$$\underline{\text{Ex 5}} \quad \text{Factor} \quad x^2 \left(\cancel{-5y} \right) x + (\cancel{6y^2})^c$$

$$\begin{array}{r}
 \textcircled{\times} +6y^2 | (2y, 3y), (-2y, -3y), (1y, 6y), (-1y, -6y) \\
 \textcircled{+} -5y | 5y \quad \boxed{-5y}
 \end{array}$$

$$= \boxed{(x-2y)(x-3y)}$$

Special Cases Difference of Squares

Notice $(a - b)(a + b)$

$$= a^2 - \cancel{ab} + \cancel{ab} - b^2$$

$$= a^2 - b^2$$

If our polynomial is

$$\underline{\text{(something)}}^2 - \underline{\text{(something)}}^2$$

we can factor it quickly

Ex 6 Factor $x^2 - 4y^2 = ((x-2y)(x+2y))$

$$\sqrt{1^{\text{st}}} = \sqrt{x^2} = x \quad \text{"a"}$$

$$\sqrt{2^{\text{nd}}} = \sqrt{4y^2} = 2y \quad \text{"b"}$$

$$x^2 - 4y^2 = (x)^2 - (2y)^2$$

"a" "b"

Ex 7 Factor $x^4 - 16$

$$\left. \begin{array}{l} \sqrt{1^{st}} = \sqrt{x^4} = x^2 \\ \sqrt{2^{nd}} = \sqrt{16} = 4 \\ \sqrt{1^{st}} = \sqrt{x^2} = x \\ \sqrt{2^{nd}} = \sqrt{4} = 2 \end{array} \right\} = (x^2 + 4)(x^2 - 4)$$
$$= (x^2 + 4)(x - 2)(x + 2)$$

$$x^2 + 4 = x^2 + 0x + 4$$

$$\begin{array}{c|cccc} (x) + 4 & (1, 4), (-1, -4), (2, 2), (-2, -2) \\ \hline (+) 0 & 5 & -5 & 4 & -4 \end{array}$$