

In general, we will factoring quadratic trinomials of the form:

$$\frac{ax^2}{1} + \frac{bx}{2} + \frac{c}{3}$$

Today, we will start by factoring $x^2 + bx + c$ ($a=1$)

$$x^2 + bx + c = (x+p)(x+q)$$

Factors

$$x^2 + bx + c = x^2 + px + qx + pq$$

$$x^2 + \underline{bx} + \underline{c} = x^2 + (\underline{p+q})x + \underline{pq}$$

In order to factor $x^2 + bx + c$ we need to find p, q so that:

$$p+q = b ; \quad p \overset{\text{mult.}}{q} = c$$

(NOTE: this ONLY works for $a=1$)

Ex 1 Factor $x^2 + \underline{6}x + \underline{8}$

$$\begin{array}{l} \otimes +8 \\ \oplus +6 \end{array} \left| \begin{array}{l} (2,4), (1,8), (-2,4), (-1,8) \\ 6 \quad 9 \end{array} \right.$$

$$x^2 + 6x + 8 = \underline{(x+2)(x+4)}$$

check $x^2 + 2x + 4x + 8$
 $= x^2 + 6x + 8$ (✓)

$$\begin{array}{l} \otimes +8 \\ \oplus +6 \end{array} \Rightarrow (2,4) \Rightarrow (x+2)(x+4)$$

Ex 2 Factor $x^2 - 15 + 2x$
 $= x^2 + 2x - 15$

$$\begin{array}{l} \otimes -15 \\ \oplus +2 \end{array} \left| \begin{array}{l} (-3,5), (-5,3), (1,-15), (-1,15) \\ 2 \end{array} \right.$$

$$\begin{aligned} x^2 + 2x - 15 &= (x-3)(x+5) \\ &= (x+5)(x-3) \end{aligned}$$

Ex 3 Factor $x^2 - 5x - 4$

$$\begin{array}{l} \otimes -4 \\ \oplus -5 \end{array} \left| \begin{array}{l} (1,-4), (-1,4), (2,-2) \\ -3 \quad 3 \quad 0 \end{array} \right.$$

None of the factors work
∴ Cannot be factored.

Ex 4 Factor $-2x^3 + 2x^2 + 24x$

$$= x(-2x^2 + 2x + 24)$$

$$= -2x(x^2 - 1x - 12)$$

$$\begin{array}{l} \otimes -12 \mid (-6, 2), (4, -3), (3, -4), (6, -2), (1, -12), (-1, 12) \\ \oplus -1 \mid -4 \quad \quad \quad 1 \quad \quad \quad -1 \end{array}$$

$-2x(x+3)(x-4)$ *check*

Ex 5 Factor $x^2 \overset{b}{(-5y)}x \overset{c}{(+6y^2)}$

$$\begin{array}{l} \otimes +6y^2 \mid (1y, 6y), (-1y, -6y), (-2y, -3y), (2y, 3y) \\ \oplus -5y \mid 1y \quad \quad \quad -1y \quad \quad \quad -5y \end{array}$$

$(x-2y)(x-3y)$

Special Cases Difference of Squares

Notice $(a-b)(a+b)$

$$= a^2 - \cancel{ab} + \cancel{ab} - b^2$$

$$= \begin{matrix} \uparrow & & \uparrow \\ a^2 & - & b^2 \\ \text{Something}^2 & & \text{Diff.} \end{matrix} \quad \text{Something}^2$$

$$\begin{array}{l} \text{Ex 6} \\ \hline \sqrt{1^{\text{st}}} \end{array} = \sqrt{x^2} = x \quad \leftarrow \text{"a"}$$

$$\sqrt{2^{\text{nd}}} = \sqrt{4y^2} = 2y \quad \leftarrow \text{"b"}$$

$$= (x-2y)(x+2y)$$

Ex 7 Factor $x^4 - 16$

$$\sqrt{1^{\text{st}}} = \sqrt{x^4} = x^2 \leftarrow a$$

$$\sqrt{2^{\text{nd}}} = \sqrt{16} = 4 \leftarrow b$$

$$x^4 - 16 = (\underline{x^2 - 4})(x^2 + 4)$$

$$\sqrt{1^{\text{st}}} = \sqrt{x^2} = x$$

$$\sqrt{2^{\text{nd}}} = \sqrt{4} = 2$$

$$x^4 - 16 = (x - 2)(x + 2)(x^2 + 4)$$