Today: Lots of notes
2.1 & 2.2

Th: Gr8 Camp (Th/Fri)
Gr10 Conference (Th/Fri)
work time

Next Tu: Go over test & 2.1/2.2

Next Th: Quiz 2.1/2.2

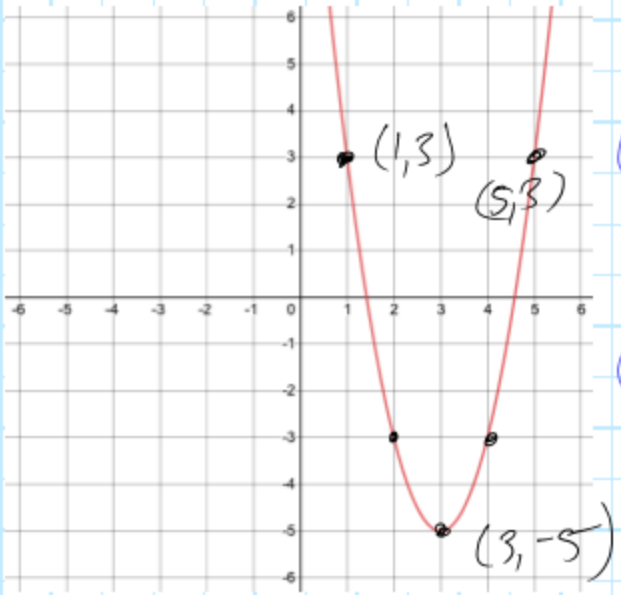
2.1 Finding Equations of Quadratics

If you are given information about a quadratic, use standard form to find equation

$$f(x) = a(x-h)^2 + k$$

Vertex: (h, k)

To get 'a', we need one other point



Ex 1 Find equation

① Vertex $(3,-5)$

$f(x) = a(x-3)^2 - 5$

② other point $(1,3)$

$f(1) = 3 = a(1-3)^2 - 5$

$3 = a(-2)^2 - 5$

$3 = 4a - 5$

$8 = 4a$ (11) $a = 2$

$f(x) = 2(x-3)^2 - 5$

Ex 2 A parabola has a Minimum at $(-4,-5)$ and goes through $(2,7)$

$a > 0$

vertex

① vertex $(-4, -5)$

$$f(x) = a(x-(-4))^2 - 5$$

$$= a(x+4)^2 - 5$$

② other point $(2, 7)$

$$f(2) = 7 = a(2+4)^2 - 5$$

$$7 = 36a - 5$$

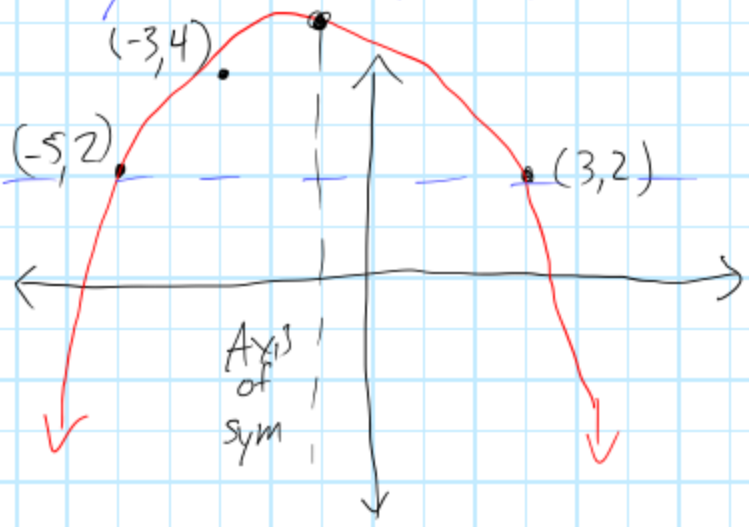
$$12 = 36a$$

$$\frac{12}{36} = a = \frac{1}{3}$$

$$\boxed{f(x) = \frac{1}{3}(x+4)^2 - 5}$$

Ex 3 A parabola passes through $(-5, 2)$, $(3, 2)$, $(-3, 4)$. Find the equation.

If you don't have the vertex, draw a picture



Down $a<0$

① Axis of Symmetry

Middle of $(-5, 2)$ and $(3, 2)$

$$= \frac{-5+3}{2} = \frac{-2}{2}$$

$$x = -1$$

$$f(x) = a(x+1)^2 + k$$

② Sub in a Mirror Point

$$f(3) = 2 = a(3+1)^2 + k$$

$$2 = 16a + k$$

ⓘ $2 - 16a = k$

(3) Sub in another point (Not mirror point)

$(-3,4)$ $f(-3) = 4 = a(-3+1)^2 + k$

$4 = 4a + k$

(ii) $4 - 4a = k$

(4) Solve both (i) and (ii)

(i) $2 - 16a = k$ (ii) $4 - 4a = k$

$2 - 16a = 4 - 4a$

$2 = 4 + 12a$

$-2 = 12a$

$\frac{-2}{12} = a = -\frac{1}{6}$

(i) $\Rightarrow \quad 2 - 16\left(-\frac{1}{6}\right) = k$

$$2 + \frac{16}{6} = k$$

$$2 + \frac{8}{3} = k$$

$$\frac{6}{3} + \frac{8}{3} = k = \frac{14}{3}$$

$$\boxed{f(x) = -\frac{1}{6}(x+1)^2 + \frac{14}{3}}$$

2.2 General to Standard form

Standard to general is easy.

$f(x) = 3(x-4)^2 - 2 \Rightarrow$ expand

$= 3(x-4)(x-4) - 2$ ← Perfect Square

$= 3(x-4)(x-4) - 2$

Perfect Square

$= 3(x^2 - 8x + 16) - 2$

$= 3x^2 - 12x - 12x + 48 - 2$

$= 3x^2 - 24x + 46$ ✓ (11)

Backwards is harder...

We need to make a perfect square in our quadratic (Completing the Square)

Ex1 Find k so our quadratic contains a Perfect Square

a) $x^2 - 18x + k$

$\sqrt{1^{st}} = \sqrt{x^2} = x$

$\sqrt{last} = \sqrt{k}$

Middle: $\pm 2(x)(\sqrt{k}) = -18x$

$\div -2 \quad \div -2$

$-2\sqrt{k} = -18$

$(\sqrt{k})^2 = +9^2$

$k = 81$

$x^2 - 18x + 81$

$= (x - 9)^2$

b) $-2x^2 - 12x + k$

$-2\left(x^2 + 6x + \frac{k}{-2}\right)$

$\sqrt{1^{st}} = \sqrt{x^2} = x$

$\sqrt{last} = \sqrt{\frac{k}{-2}}$

Middle: $\pm 2(x)\left(\sqrt{\frac{k}{-2}}\right) = +6x$

$2\sqrt{\frac{k}{-2}} = 6$

$\sqrt{\frac{k}{-2}} = 3$

$\frac{k}{-2} = 9$

$k = -18$

$k = -18$

$$-2\left(x^2 + 6x + \frac{-18}{-2}\right)$$

$$= -2(x^2 + 6x + 9)$$

$$= -2(x + 3)^2$$

Ex 2 Complete the Square to Convert to general form

a) $f(x) = [x^2 - 10x] + 22$ ← ignore

perfect Square

$$= [(x^2 - 10x + k) - k] + 22$$

$\sqrt{1^{st}} = \sqrt{x^2} = x$

$\sqrt{last} = \sqrt{k}$

Middle: $\pm 2(x)(\sqrt{k}) = \ominus 10x$

$\div 2$

$+2\sqrt{k} = +10 \quad \div 2$

$(\sqrt{k})^2 = 5^2$

$k = 25$

Short Cut

$(\text{middle} \div 2)^2$

$= (-10 \div 2)^2$

$= (-5)^2$

$= 25$

$f(x) = [(x^2 - 10x + 25) - 25] + 22$

$= [(x-5)^2 - 25] + 22$

$= (x-5)^2 - 3$

Vertex:

$(5, -3)$

x-ints: $(x-5)^2 - 3 = 0$

$(x-5)^2 = 3$

$x-5 = \pm\sqrt{3}$

$x = 5 \pm \sqrt{3}$

$(5+\sqrt{3}, 0)$

$(5-\sqrt{3}, 0)$

↑ Hard to get from factoring

b) $f(x) = [-2x^2 - 3x] - 3$ ← ignore

$= -2\left[x^2 + \frac{3}{2}x\right] - 3$

$$= -2\left[\left(x^2+\frac{3}{2}x+k\right)-k\right]-3$$

Shortcut: middle ÷ 2

$$\frac{3}{2} \div \frac{2}{1} = \frac{3}{2} \times \frac{1}{2}$$

$$= \frac{3}{4}$$

$$k = (\text{middle} \div 2)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$= -2\left[\left(x^2+\frac{3}{2}x+\frac{9}{16}\right)-\frac{9}{16}\right]-3$$

$$= -2\left[\left(x+\frac{3}{4}\right)^2-\frac{9}{16}\right]-3$$

$$= -2\left(x+\frac{3}{4}\right)^2+\frac{9}{8}-3$$

$$= -2\left(x+\frac{3}{4}\right)^2+\frac{9}{8}-\frac{24}{8}$$

$$= \boxed{-2\left(x+\frac{3}{4}\right)^2-\frac{15}{8}} \checkmark$$