Finding Quadratic Equations + General to Standard Form

Wednesday, October 2, 2019 8:02 AM

Today: Lots of notes 2.1/2.2

Thurs: Work @ Conference (6/8 Camp)

next TU: Go over test and 2.1/2.2

next Th: 2.1/2.2 Quiz

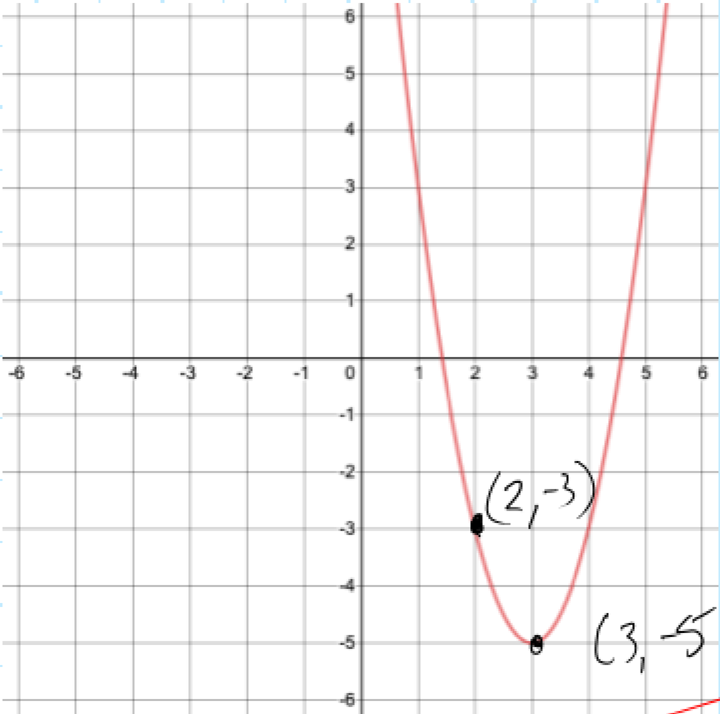
2.1 Finding Quadratic Equations

If you have information about a quadratic and you want to find the equation $\Rightarrow$ use standard form

$$f(x) = a(x-h)^2 + k$$

vertex: (h, k)

To get 'a', need one other point



Ex 1 Find Equation

① vertex $(3, -5)$

$f(x) = a(x-3)^2 - 5$

$a > 0$

② other point

$(2, -3)$

$f(2) = -3 = a(2-3)^2 - 5$

$-3 = a(-1)^2 - 5$

$-3 = 1a - 5$

$2 = a$

$$\boxed{f(x) = 2(x-3)^2 - 5}$$

Ex2 A quadratic has a minimum at $(-4, -5)$ and passes through $(2, 7)$. Find the equation.

$a > 0$ vertex

① vertex $(-4, -5)$

$$f(x) = a(x-(-4))^2 - 5$$

$$f(x) = a(x+4)^2 - 5$$

② other point $(2, 7)$

$$f(2) = 7 = a(2+4)^2 - 5$$

$$7 = 36a - 5$$

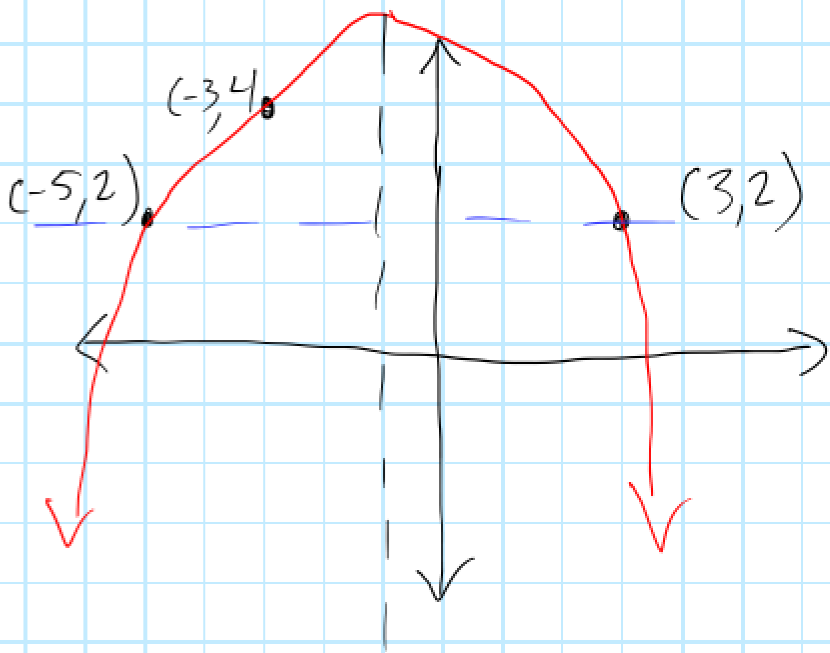
$$12 = 36a$$

$$\frac{12}{36} = a = \frac{1}{3}$$

$$\boxed{f(x) = \frac{1}{3}(x+4)^2 - 5}$$

Ex 3 A parabola passes through $(-5,2)$, $(3,2)$, $(-3,4)$. Find equation.

If you don't have vertex, draw a picture



① Down $a<0$

② Axis of Symmetry

Middle of $(-5,2)$ and $(3,2)$

$$= \frac{-5+3}{2} = \frac{-2}{2}$$

$$x = -1$$

$f(x) = a(x-(-1))^2$

$x = -1$

$$f(x) = a(x-(-1))^2 + k$$
$$= a(x+1)^2 + k$$

② Sub in a point (mirror point)

$(3, 2)$ $\quad f(3) = 2 = a(3+1)^2 + k$

$$2 = 16a + k$$

ⓘ $2 - 16a = k$

③ Sub in another point (NOT mirror point)

$(-3, 4) \Rightarrow f(-3) = 4 = a(-3+1)^2 + k$

$$4 = 4a + k$$

ⓘⓘ $4 - 4a = k$

4 use both i and ii to solve for a, k

i $2-16a=k$ ii $4-4a=k$

$$2-16a=4-4a$$

$+16a$ $+16a$

$$2=4+12a$$

-4 -4

$$-2=12a$$

$$-\frac{1}{6}=a$$

ii $4-4\left(-\frac{1}{6}\right)=k$

$$4+\frac{4}{6}=k=4+\frac{2}{3}=\frac{12}{3}+\frac{2}{3}$$

$$k=\frac{14}{3}$$

$$\boxed{f(x)=-\frac{1}{6}(x+1)^2+\frac{14}{3}}$$

2.2 General to Standard form

Standard to general is easy.

$$f(x) = 3(x-4)^2 - 2 \Rightarrow \text{expand}$$

Perfect Square

$$= 3(x-4)(x-4) - 2$$

$$= 3(x^2 - 8x + 16) - 2$$

$$= 3x^2 - 12x - 12x + 48 - 2$$

$$= 3x^2 - 24x + 46$$

Going backwards is harder...

To go backwards, we need to make a perfect square (Completing the Square)

Ex 1 Find k so that the trinomial contains a perfect square

a) $x^2 - 16x + k$

$\sqrt{1^{st}} = \sqrt{x^2} = x$

$\sqrt{last} = \sqrt{k}$

Middle: $\pm 2(\cancel{x})(\sqrt{k}) = -16\cancel{x}$

$-2\sqrt{k} = -16$

$(\sqrt{k})^2 = 8^2$

$k = 64$

$x^2 - 16x + 64$

$= (x - 8)^2$

b) $-2x^2 + 12x + k$

$-2\left(x^2 - 6x + \frac{k}{-2}\right)$

$\sqrt{1^{st}} = \sqrt{x^2} = x$

$\sqrt{last} = \sqrt{\frac{k}{-2}}$

Middle: $\pm 2(\cancel{x})\left(\sqrt{\frac{k}{-2}}\right) = -6\cancel{x}$

$-2\sqrt{\frac{k}{-2}} = -6$

$\sqrt{\frac{k}{-2}} = 3$

$\frac{k}{-2} = 9$

$= (x - 8)$

$\frac{k}{-2} = 9$

$k = -18$

$$-2\left(x^2 - 6x + \frac{-18}{-2}\right)$$

$$-2\left(x^2 - 6x + 9\right)$$

$$= -2(x - 3)^2$$

Ex 3 Complete the Square to convert to standard form

a) $f(x) = [x^2 + 10x] + 22$ ← ignore

$= [(x^2 + 10x + k) - k] + 22$

perfect square

$\sqrt{1^{st}} = \sqrt{x^2} = x$

$\sqrt{\text{last}} = \sqrt{k}$

middle: $\pm 2(\cancel{x})(\sqrt{k}) = 10\cancel{x}$

$2\sqrt{k} = 10$

$\div 2$

$\sqrt{k} = 5$

Square

$k = 25$

Short cut

$(\text{middle} \div 2)^2$

$= (10 \div 2)^2$

$= 5^2$

$= 25$

$f(x) = [(x^2 + 10x + 25) - 25] + 22$

$$= \left[(x+5)^2 - 25\right] + 22$$

middle $\div$ 2

$$\boxed{f(x) = (x+5)^2 - 3}$$

vertex: $(-5, -3)$

x-ints: $(x+5)^2 - 3 = 0$

$$(x+5)^2 = 3$$

$$x+5 = \pm\sqrt{3}$$

$$x = -5 \pm \sqrt{3}$$

Can't get easily by factoring

$$(-5+\sqrt{3}, 0), \ (-5-\sqrt{3}, 0)$$

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b) $f(x) = \left[-2x^2 - 3x\right] - 3$ ← ignore

$$= -2\left[1x^2 + \frac{3}{2}x\right] - 3$$

$$= -2\left[\left(x^2+\frac{3}{2}x+k\right)-k\right]-3$$

<u>Shortcut:</u>

$$\text{middle} \div 2 = \frac{3}{2} \div \frac{2}{1} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$$

$$k = (\text{middle} \div 2)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$= -2\left[\left(x^2+\frac{3}{2}x+\frac{9}{16}\right)-\frac{9}{16}\right]-3$$

$$= -2\left[\left(x+\frac{3}{4}\right)^2-\frac{9}{16}\right]-3$$

$$= -2\left(x+\frac{3}{4}\right)^2+(-2)\left(-\frac{9}{16}\right)-3$$

$$= -2\left(x+\frac{3}{4}\right)^2+\frac{9}{8}-3$$

$$= -2\left(x+\frac{3}{4}\right)^2 + \frac{9}{8} - \frac{24}{8}$$

$$\boxed{= -2\left(x+\frac{3}{4}\right)^2 - \frac{15}{8}}$$

vertex:

$\left(-\frac{3}{4}, -\frac{15}{8}\right)$

x-ints:

$$-2\left(x+\frac{3}{4}\right)^2 - \frac{15}{8} = 0$$

$$-2\left(x+\frac{3}{4}\right)^2 = \frac{15}{8}$$

$$\left(x+\frac{3}{4}\right)^2 = -\frac{15}{16}$$

$$x+\frac{3}{4} = \pm\sqrt{-\frac{15}{16}}$$

No root

No x-ints