Today: Lots of notes
(2.1 / 2.2)

Thurs: Away Gr 8 Camp
work time

next Tu: Go over test / 2.1 / 2.2

Next Th: 2.1 / 2.2 quiz

2.1) Finding quadratic equations

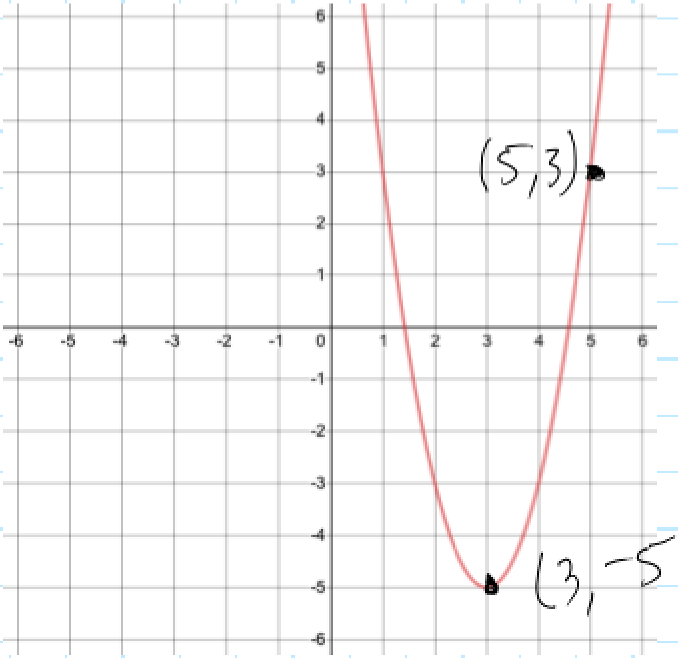
If you have information about a quadratic and you want to find the equation ⇒ use standard form

$$f(x) = a(x-h)^2 + k \qquad (h, k)$$

vertex

If you know vertex ⇒ h, k ✓

If you know another point ⇒ a ✓



Ex 1 Find equation

1 vertex

$(3,-5)$

$$f(x) = a(x-3)^2 - 5$$

$$a > 0$$

2 other point

$(5,3)$

$$f(5) = 3 = a(5-3)^2 - 5$$

$$3 = a(2)^2 - 5$$

$$\overset{+5}{3} = 4a - \overset{+5}{5} \Rightarrow \overset{\div 4}{8} = 4a^{\div 4}$$

$$a = 2$$

$$\boxed{f(x) = 2(x-3)^2 - 5}$$

Ex 2 A quadratic passes through $(2,7)$ and has a minimum at $(-4,-5)$. Find equation.

up

vertex

① vertex $(-4,-5)$

$$f(x) = a(x-(-4))^2 - 5 = a(x+4)^2 - 5$$

② Other point $(2,7)$

$$f(2) = 7 = a(2+4)^2 - 5$$

$$7 = a(6)^2 - 5$$

$$7 = 36a - 5$$

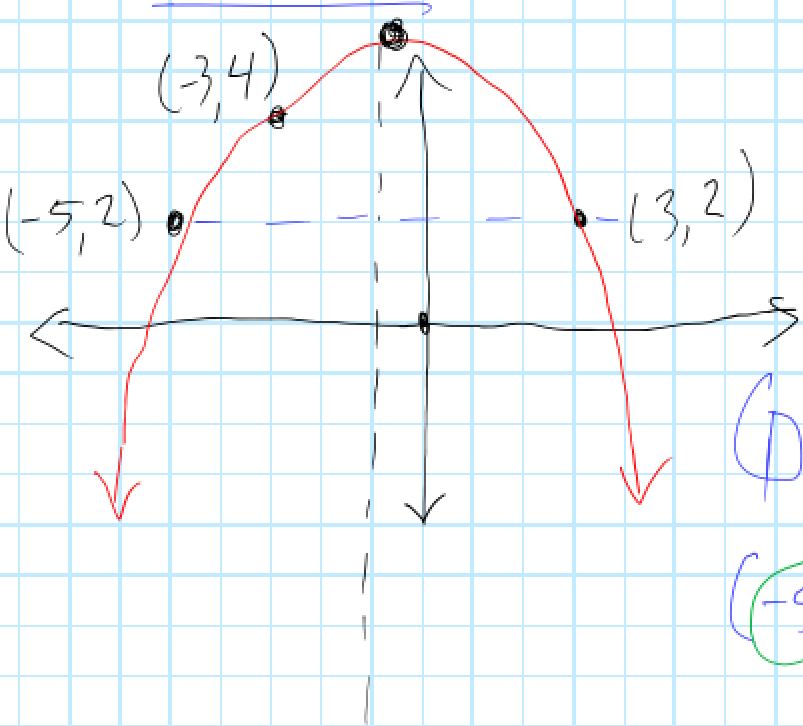
$$12 = 36a$$

$$\frac{12}{36} = a = \frac{1}{3}$$

$$\boxed{f(x) = \frac{1}{3}(x+4)^2 - 5}$$

Ex 3 A parabola passes through $(-5, 2)$, $(3, 2)$, $(-3, 4)$. Find equation.

If no info about vertex, draw picture!



① opens down $a < 0$

② Axis of Symmetry

① Middle of $(-5, 2)$ and $(3, 2)$

$$= \frac{-5+3}{2} = \frac{-2}{2}$$

$$x = -1$$

$$f(x) = a(x-(-1))^2 + k$$

$x = -1$

$f(x) = a(x-(-1))^2 + k$

$f(x) = a(x+1)^2 + k$

② sub in one point (mirror points)

$(3,2) \Rightarrow f(3) = 2 = a(3+1)^2 + k$

$2 = 16a + k$

ⓘ $2 - 16a = k$

③ Sub in other point (NOT mirror point)

$(-3,4) \Rightarrow f(-3) = 4 = a(-3+1)^2 + k$

$$4 = 4a + k$$

ii $$4 - 4a = k$$

4 Substitute and solve.

i $2 - 16a = k$ ii $4 - 4a = k$

$$2 - \cancel{16a}^{+16a} = 4 - 4a^{+16a}$$

$$\overset{-4}{2} = \overset{-4}{4} + 12a$$

$$\div 12 \quad -2 = 12a \div 12$$

$$-\frac{1}{6} = a$$

i) $2 - 16\left(-\frac{1}{6}\right) = k$

$2 + \frac{16}{6} = k =$

$2 + \frac{8}{3} = k = \frac{6}{3} + \frac{8}{3} = \frac{14}{3} = k$

$$\boxed{f(x) = -\frac{1}{6}(x+1)^2 + \frac{14}{3}}$$

$\boxed{2.2}$ General to Standard Form

Standard to general is easy.

Ex $f(x) = 3(x-4)^2 - 3 \Rightarrow$ expand

Perfect Square

$$= 3(x-4)(x-4) - 3$$

$$= 3(x^2 - 8x + 16) - 3$$

$$= 3x^2 - 12x - 12x + 48 - 3$$

$$= 3x^2 - 24x + 45 \checkmark$$

But how do we go backwards?

In order to go backwards, we need to make a perfect square (Completing the square).

Ex 1 Find k so that the trinomial contains a perfect square

a) $x^2 + 16x + k$

$\sqrt{1^{st}} = \sqrt{x^2} = x$

$\sqrt{last} = \sqrt{k}$

Middle: $\pm 2(\cancel{x})(\sqrt{k}) = 16\cancel{x}$

$\div 2 \quad 2\sqrt{k} = 16 \quad \div 2$

$(\sqrt{k})^2 = (8)^2$

$k = 64$

$x^2 + 16x + 64$

$= (x+8)^2$

b) $-2x^2 - 12x + k$

$-2\left(x^2 + 6x + \frac{k}{-2}\right)$

$\sqrt{1^{st}} = \sqrt{x^2} = x$

$\sqrt{last} = \sqrt{\frac{k}{-2}}$

Middle: $\pm 2(\cancel{x})\left(\sqrt{\frac{k}{-2}}\right) = 6\cancel{x}$

$2\sqrt{\frac{k}{-2}} = 6$

$\sqrt{\frac{k}{-2}} = 3$

$$\frac{k}{-2} = 9$$

$$\underline{k = -18}$$

$$-2\left(x^2 + 6x + \frac{-18}{-2}\right)$$

$$-2(x^2 + 6x + 9)$$

$$-2(x+3)^2$$

<u>Ex</u> 1 Complete the Square to convert to standard form

a)

$$f(x) = [x^2 + 10x] + 22$$ ← ignore

$$= [(x^2 + 10x + k) - k] + 22$$

perfect square

perfect square

$$\sqrt{1st} = \sqrt{x^2} = x$$

$$\sqrt{last} = \sqrt{k}$$

middle ÷ 2 then square it

Middle: $\pm 2(\cancel{x})(\sqrt{k}) = 10\cancel{x}$

$$2\sqrt{k} = 10$$

$$\sqrt{k} = 5 \Rightarrow k = 25$$

$$f(x) = \left[(x^2 + 10x + 25) - 25\right] + 22$$

$$= \left[(x+5)^2 - 25\right] + 22$$

$$= (x+5)^2 - 3 \quad \checkmark$$

Vertex: $(-5, -3)$ ✓

x-ints : $(x+5)^2 - 3 = 0$

$$(x+5)^2 = 3$$

$$x+5 = \pm\sqrt{3}$$

$$x = -5 \pm \sqrt{3}$$

not easy to get by factoring

$$(-5+\sqrt{3}, 0), \quad (-5-\sqrt{3}, 0)$$

b) $f(x) = \left[-2x^2 - 3x\right] - 3$ ← ignore

$$= -2\left[x^2 + \frac{3}{2}x\right] - 3$$

$$= -2\left[\left(x^2 + \frac{3}{2}x + k\right) - k\right] - 3$$

perfect square

perfect Square

$\sqrt{1^{st}} = \sqrt{x^2} = x$

$\sqrt{last} = \sqrt{k}$

Middle: $\pm 2(\cancel{x})(\sqrt{k}) = \frac{3}{2}\cancel{x}$

$2\sqrt{k} = \frac{3}{2}$

$\sqrt{k} = \frac{3}{4} \Rightarrow k = \frac{9}{16}$

$(\text{Middle} \div 2)^2$

$= \left(\frac{3}{2} \div 2\right)^2$

$= \left(\frac{3}{4}\right)^2$

$= \frac{9}{16}$

$= -2\left[\left(x^2 + \frac{3}{2}x + \frac{9}{16}\right) - \frac{9}{16}\right] - 3$

$= -2\left[\left(x + \frac{3}{4}\right)^2 - \frac{9}{16}\right] - 3$

$$= -2\left(x+\frac{3}{4}\right)^2 + (-2)\left(-\frac{9}{16}\right) - 3$$

$$= -2\left(x+\frac{3}{4}\right)^2 + \frac{9}{8} - 3$$

$$= -2\left(x+\frac{3}{4}\right)^2 + \frac{9}{8} - \frac{24}{8}$$

$$= -2\left(x+\frac{3}{4}\right)^2 - \frac{15}{8}$$

<u>vertex</u>: $\left(-\frac{3}{4}, -\frac{15}{8}\right)$

<u>x ints</u>: $-2\left(x+\frac{3}{4}\right)^2 - \frac{15}{8} = 0$

$$-2\left(x+\frac{3}{4}\right)^2 = \frac{15}{8}$$

$$\left(x+\frac{3}{4}\right)^2 = -\frac{15}{16}$$

$$x+\frac{3}{4} = \pm\sqrt{-\frac{15}{16}}$$ ← <u>NO root</u>

<u>NO x-ints</u>