Warm-Up

Find the vertex and all intercepts by completing the square then graph:

$$f(x) = \left[-\frac{1}{2}x^2 + 3x\right] - 2 \quad \text{ignore}$$

$$= -\frac{1}{2}\left[\frac{\cancel{-\frac{1}{2}}x^2}{\cancel{-\frac{1}{2}}} + \frac{3x}{-\frac{1}{2}}\right] - 2$$

$$3 \div -\frac{1}{2} = 3 \cdot -\frac{2}{1}$$

$$= -\frac{1}{2}\left[(x^2 - 6x + k) - k\right] - 2$$

$$\left(\frac{1}{2}\text{middle}\right)^2 = \left(\frac{1}{2}(-6)\right)^2$$

$$= (-3)^2 = 9$$

$$= -\frac{1}{2}\left[(x^2 - 6x + 9) - 9\right] - 2$$

$$= -\frac{1}{2}\left[(x-3)^2 - 9\right] - 2$$

$$= -\frac{1}{2}(x-3)^2 + \left(-\frac{1}{2}\right)(-9) - 2$$

$$= -\frac{1}{2}(x-3)^2 + \frac{9}{2} - \frac{4}{2}$$

$$= -\frac{1}{2}(x-3)^2 + \frac{5}{2}$$

vertex: $\left(3, \frac{5}{2}\right)$

y-int: $f(0) = -\frac{1}{2}(0-3)^2 + \frac{5}{2} = -\frac{1}{2}(-3)^2 + \frac{5}{2}$

$(0, -2)$ $\quad = -\frac{9}{2} + \frac{5}{2} = -\frac{4}{2} = -2$

x int: $f(x) = 0 = -\frac{1}{2}(x-3)^2 + \frac{5}{2}$

$-2x \quad -\frac{5}{2} = \overset{x-2}{-\frac{1}{2}}(x-3)^2$

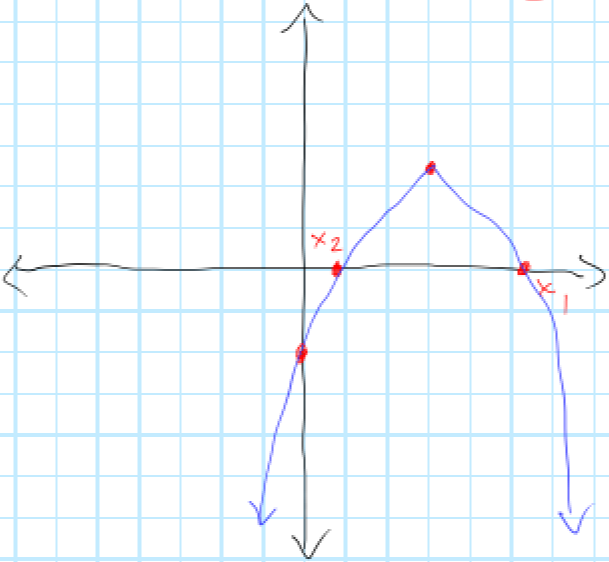
$5 = (x-3)^2$

$\pm\sqrt{5} = x - 3$

$3 \pm \sqrt{5} = x$

$(3 \pm \sqrt{5}, 0)$ or $(3+\sqrt{5}, 0), (3-\sqrt{5}, 0)$

For graphing $\approx (5.2, 0)$, $(0.8, 0)$
$\quad x_1 \qquad x_2$



A rancher is setting up a fenced rectangular area for her sheep next to a cliff (no fencing needed next to the cliff). If she has 1000m of fencing in total to use and wants to enclose the largest area: $ax^2 + bx + c$

a) What dimensions should the rectangle be?

b) What is the total enclosed area?

① Draw picture

② variables/ equation

$$A = w \cdot l = w \cdot (1000 - 2w) = 1000w - 2w^2 = A$$

$$1000 = 2w + l$$

$$1000 - 2w = l$$

③ vertex $a = -2$ $b = 1000$ $c = 0$

$$h = \frac{-b}{2a} = \frac{-(1000)}{2(-2)} = 250$$

x value vertex ⇒ width at max

a) width: 250m

length: $1000 - 2(250) = 500$m

$\boxed{250\text{m} \times 500\text{m}}$

b) $A = l \cdot w = 500 \cdot 250 = \boxed{125\,000\text{ m}^2}$ max y value max area

Ex 2

A hotel is trying to decide on a nightly rate to maximize income. When they charge \$80 a night, 75% of the 400 rooms are booked. A survey of customers shows that increasing the price by \$5 would decrease the number of customers by 10. Find the nightly rate and number of customers that maximizes their income.

① variables/ equation

Income = #rooms · price

Start: 75% of 400

$400 \cdot 0.75 = 300$

x = # of times we increase price

$$I = (300 - 10x) \cdot (80 + 5x)$$

$$I = 24000 - 800x + 1500x - 50x^2$$

$$= 24000 + 700x - 50x^2$$

Vertex: $a = -50$

$b = 700$

$c = 24000$

$$h = \frac{-b}{2a} = \frac{-700}{2(-50)} = 7$$

x value of vertex ⇒ max

max Income by increasing 7 times

Rate: $80 + 5(7) = \$115$ People: $300 - 10(7) = 230$

Ex 3

b) How high does the apple go?
c) When does the apple reach its maximum height?
d) When does it hit the ground?

a) $t=0 \Rightarrow H(0) = -5(0)^2 + 12(0) + 11 = \underline{11\ m}$

b) max height $\Rightarrow$ $\underline{\text{vertex}}$: $h = \frac{-b}{2a} = \frac{-12}{2(-5)}$ $a=-5$, $b=12$, $c=11$

x val vertex $\Rightarrow$ t @ max $= \frac{12}{10} = \frac{6}{5} = \underline{1.2s}$

$H\left(\frac{6}{5}\right) = -5\left(\frac{6}{5}\right)^2 + 12\left(\frac{6}{5}\right) + 11$

$= -\cancel{5}\left(\frac{36}{\cancel{25}_{5}}\right) + \frac{72}{5} + 11 = -\frac{36}{5} + \frac{72}{5} + \frac{55}{5}$

$= \frac{91}{5} = \boxed{18.2m}$ max height

c) $\underline{t = 1.2s}$

d) $H(t) = 0 = -5t^2 + 12t + 11$

$0 = -5(x - \underbrace{\tfrac{6}{5}}_{h})^2 + \underbrace{\tfrac{91}{5}}_{k}$

$-\frac{91}{5} = -5\left(x - \frac{6}{5}\right)^2$

$\frac{91}{25} = \left(x - \frac{6}{5}\right)^2$

$\pm\sqrt{\frac{91}{25}} = x - \frac{6}{5} \Rightarrow x = \frac{6}{5} \pm \sqrt{\frac{91}{25}}$

$x = \frac{6}{5} + \sqrt{\frac{91}{25}} = \boxed{3.1s}$

$x = \frac{6}{5} - \sqrt{\frac{91}{25}} = \cancel{-0.71s}$