Warm-UP

Find the vertex and all intercepts by completing the Square then graph:

$$f(x) = \left[-\frac{1}{2}x^2 + 3x\right] - 2 \quad \text{ignore}$$

$$= -\frac{1}{2}\left[\frac{-\frac{1}{2}x^2}{-\frac{1}{2}} + \frac{3x}{-\frac{1}{2}}\right] - 2 \qquad 3 \div -\frac{1}{2} = 3 \times -\frac{2}{1}$$

$$= -\frac{1}{2}\left[(x^2 - 6x + k) - k\right] - 2$$

$$\left(\tfrac{1}{2}\text{middle}\right)^2 = \left(\tfrac{1}{2}(-6)\right)^2 = (-3)^2 = 9$$

$$= -\frac{1}{2}\left[(x^2 - 6x + 9) - 9\right] - 2$$

$$= -\frac{1}{2}\left[(x-3)^2 - 9\right] - 2$$

$$= -\frac{1}{2}(x-3)^2 + \left(-\tfrac{1}{2}\right)(-9) - 2$$

$$= -\frac{1}{2}(x-3)^2 + \frac{9}{2} - \frac{4}{2}$$

$$= -\frac{1}{2}(x-3)^2 + \frac{5}{2}$$

Vertex: $(3, \frac{5}{2})$

y int: $f(0) = -\frac{1}{2}(0-3)^2 + \frac{5}{2}$

$= -\frac{1}{2}(-3)^2 + \frac{5}{2} = -\frac{9}{2} + \frac{5}{2}$

$= -\frac{4}{2} = -2$ $(0, -2)$

x ints: $-\frac{1}{2}(x-3)^2 + \frac{5}{2} = 0$

$-2 \cdot -\frac{1}{2}(x-3)^2 = -\frac{5}{2} \cdot -2$

$(x-3)^2 = 5$

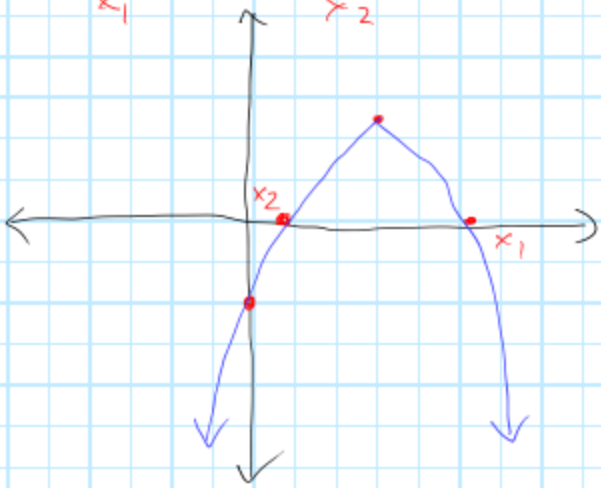
$x - 3 = \pm\sqrt{5}$

$x = 3 \pm \sqrt{5}$

$(3+\sqrt{5}, 0)$, $(3-\sqrt{5}, 0)$ or $(3 \pm \sqrt{5}, 0)$

For graph $(5.2, 0)$, $(0.8, 0)$

x_1 x_2



Ex 1

A rancher is setting up a fenced rectangular area for her sheep next to a cliff (no fencing needed next to the cliff). If she has 1000m of fencing in total to use and wants to enclose the largest area:

a) What dimensions should the rectangle be?

b) What is the total enclosed area?

$ax^2 + bx + c$

① Draw picture

cliff

$Area = l \cdot w = (1000 - 2w) \cdot w = 1000w - 2w^2$

$1000 = 2w + l$

$1000 - 2w = l$

② variables

③ equation

④ Vertex $a = -2$ $b = 1000$ $c = 0$

$h = \frac{-b}{2a} = \frac{-1000}{2(-2)} = 250$ ← x value vertex

width at vertex ⇒ max

a) width: 250m

length: $1000 - 2(250) = 500$m

250m × 500m

b) $A = l \cdot w = 250 \times 500 = 125{,}000\ m^2$

Ex 2

A hotel is trying to decide on a nightly rate to maximize income. When they charge $80 a night, 75% of the 400 rooms are booked. A survey of customers shows that increasing the price by $5 would decrease the number of customers by 10. Find the nightly rate and number of customers that maximizes their income.

① Draw picture

Income = # of people • price

Start: 75% of 400

$400 \times 0.75 = 300$

② Variable

x = # of times we change price/ # of customers

$I = (300 - 10x)(80 + 5x)$

Vertex: $I = 24000 - 800x + 1500x - 50x^2$

$I = 24000 + 700x - 50x^2$

③ Equation

$a = -50$

$b = 700$

$c = 24000$

$h = \frac{-b}{2a} = \frac{-700}{2(-50)} = 7$ ← x value vertex ⇒ max

↑ increase price 7 times

rate: $80 + 5(7) = 80 + 35 = \$115$

customers: $300 - 10(7)$

$= 300 - 70$

$= 230$

Ex 3

Mr. G goes on top of Lord Byng and throws an apple upwards over the edge of the building. The height of the apple ~~rock~~ above the ground (H, in metres) is given by: $H(t) = -5t^2 + 12t + 11$, where t is the time in seconds

d) When does it hit the ground?

a) $t=0 \quad H(0) = -5(0)^2 + 12(0) + 11 = \underline{11\text{ m}}$

b) vertex: $a = -5 \quad b = 12 \quad c = 11$

$$h = \frac{-b}{2a} = \frac{-12}{2(-5)} = \frac{12}{10} = \frac{6}{5} = 1.2$$ x value vertex

$\Rightarrow$ time at max

$$k = H\left(\frac{6}{5}\right) = -5\left(\frac{6}{5}\right)^2 + 12\left(\frac{6}{5}\right) + 11$$

$$= -5\left(\frac{36}{25}\right) + \frac{72}{5} + 11 = -\frac{36}{5} + \frac{72}{5} + \frac{55}{5}$$

$$= \frac{91}{5} = \boxed{18.2\text{m}} \leftarrow \text{max height}$$

c) $\underline{t = 1.2\text{s}}$

d) $H(t) = 0 = -5t^2 + 12t + 11$

$$0 = -5\left(t - \overset{h}{\frac{6}{5}}\right)^2 + \overset{k}{\frac{91}{5}}$$

$$-\frac{91}{5} = -5\left(t - \frac{6}{5}\right)^2$$

$$\frac{91}{25} = \left(t - \frac{6}{5}\right)^2 \Rightarrow \pm\sqrt{\frac{91}{25}} = t - \frac{6}{5}$$

$$t = \frac{6}{5} \pm \sqrt{\frac{91}{25}}$$

$$t_1 = \frac{6}{5} + \sqrt{\frac{91}{25}} = \boxed{3.1\text{ s}}$$

$$t_2 = \frac{6}{5} - \sqrt{\frac{91}{25}} = \cancel{-0.71\text{ s}}$$