Warm-Up

1. Find the vertex by completing the square

$f(x) = [-x^2 + 6x] - 27$

$-\left[(x^2 - 6x + k) - k\right] - 27$

$\left(\frac{1}{2}\text{middle}\right)^2 = \left(\frac{1}{2}(-6)\right)^2$

$= (-3)^2 = 9$

$= -\left[(x^2 - 6x + 9) - 9\right] - 27$

$= -\left[(x - 3)^2 - 9\right] - 27$

$= -(x-3)^2 + 9 - 27$

$= -(x-3)^2 - 18$

$a(x-h)^2 + k$

$h = 3, \; k = -18$

$\boxed{(3, -18)}$

② Find the x-intercepts by completing the square

$$f(x) = \left[4x^2 + 7x\right] + \frac{3}{2}$$

$$= 4\left[\left(x^2 + \frac{7}{4}x + k\right) - k\right] + \frac{3}{2}$$

$$\left(\frac{1}{2}\text{middle}\right)^2 = \left(\frac{1}{2} \cdot \frac{7}{4}\right)^2$$

$$= \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

$$= 4\left[\left(x^2 + \frac{7}{4}x + \frac{49}{64}\right) - \frac{49}{64}\right] + \frac{3}{2}$$

$$= 4\left[\left(x + \frac{7}{8}\right)^2 - \frac{49}{64}\right] + \frac{3}{2}$$

$$= 4\left(x + \frac{7}{8}\right)^2 - \frac{49}{16} + \frac{3^{\times 8}}{2_{\times 8}}$$

$$= 4\left(x + \frac{7}{8}\right)^2 - \frac{49}{16} + \frac{24}{16}$$

$$= 4\left(x + \frac{7}{8}\right)^2 - \frac{25}{16} = 0$$

$\div 4 \quad 4\left(x+\frac{7}{8}\right)^2 = \frac{25}{16} \quad \div 4$

$$\left(x+\frac{7}{8}\right)^2 = \frac{25}{64}$$

$$x+\frac{7}{8} = \pm\sqrt{\frac{25}{64}}$$

$$x+\frac{7}{8} = \pm\frac{5}{8}$$

$$x = -\frac{7}{8} \pm \frac{5}{8}$$

$$x_1 = -\frac{7}{8} + \frac{5}{8} = -\frac{2}{8} = -\frac{1}{4}$$

$$x_2 = -\frac{7}{8} - \frac{5}{8} = -\frac{12}{8} = -\frac{3}{2}$$

$$\left(-\frac{1}{4}, 0\right), \left(-\frac{3}{2}, 0\right)$$

Vertex Shortcut (2.3)

To get the shortcut, we need to complete the square with variables instead of numbers.

$$f(x) = [ax^2 + bx] + c$$

$$= a\left[\left(x^2 + \frac{b}{a}x + k\right) - k\right] + c$$

$$\left(\frac{1}{2}\text{middle}\right)^2 = \left(\frac{1}{2}\frac{b}{a}\right)^2$$

$$= \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$= a\left[\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a^2}\right] + c$$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right] + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 \cancel{a}}{4a^{\cancel{2}}} + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

$$a(x-h)^2 + k \qquad \text{Vertex } (h, k)$$

$$* \boxed{h = -\frac{b}{2a}} * \qquad \boxed{k = -\frac{b^2}{4a} + c}$$

Note: 'a' is the same in general and standard form

Ex 1 Find the vertex, y-intercept and x-intercepts for $f(x) = -3x^2 + 6x + 4$

$ax^2 + bx + c$

$a = -3$

$b = 6$

$c = 4$

vertex: (h,k)

$$\boxed{h = \frac{-b}{2a}}$$

$$= \frac{-6}{2(-3)} = \frac{-6}{-6} = \underline{1}$$

x value vertex

① $\boxed{k = -\frac{b^2}{4a} + c}$

$$k = -\frac{6^2}{4(-3)} + 4$$

$$= \frac{-36}{-12} + 4$$

$$= 3 + 4 = \underline{7}$$

② $k = f(1) = -3(1)^2 + 6(1) + 4$

$$= -3 + 6 + 4$$

$$= \underline{7}$$

$$\boxed{(1, 7)}$$

y-int: $f(0) = \cancel{-3(0)^2} + \cancel{6(0)} + 4$

$$= 4$$

x-ints: use standard form

$$f(x) = a(x-h)^2 + k$$

$$= -3(x-1)^2 + 7 = 0$$

$$-3(x-1)^2 = -7$$

$$(x-1)^2 = \frac{7}{3}$$

$$x - 1 = \pm\sqrt{\frac{7}{3}}$$

$$x = 1 \pm \sqrt{\frac{7}{3}}$$

$$x_1 = 1 + \sqrt{\frac{7}{3}} \approx 2.53$$

$$x_2 = 1 - \sqrt{\frac{7}{3}} \approx -0.53$$

For the graph

$$\boxed{\left(1+\sqrt{\tfrac{7}{3}}, 0\right), \left(1-\sqrt{\tfrac{7}{3}}, 0\right)}$$