Vertex Short-Cut (2.3)

Tuesday, October 15, 2019 3:17 PM

(2.1/2.2 Quiz next class)

Warm-Up

① Find the vertex by completing the square

$f(x) = (-x^2 + 6x) - 27$

$= -\left[(x^2 - 6x + k) - k\right] - 27$

$\left(\frac{\text{middle}}{2}\right)^2 = \left(\frac{-6}{2}\right)^2$

$= (-3)^2 = 9$

$= -\left[(x^2 - 6x + 9) - 9\right] - 27$

$= -\left[(x-3)^2 - 9\right] - 27$

$= -(x-3)^2 + 9 - 27$

$= -(x-3)^2 - 18$

$a(x-h)^2 + k$

$h = 3 \quad k = -18$

$(3, -18)$

② Find
by Co
f(x)
=
=
=
=
=
=
-

(2) Find the x-intercepts by completing the square

$f(x) = (4x^2 + 7x) + \frac{3}{2}$

$= 4\left[\left(x^2 + \frac{7}{4}x + k\right) - k\right] + \frac{3}{2}$

$\left(\frac{\text{middle}}{2}\right)^2 = \left(\frac{7}{4} \div \frac{2}{1}\right)^2$

$= \left(\frac{7}{4} \times \frac{1}{2}\right)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

$= 4\left[\left(x^2 + \frac{7}{4}x + \frac{49}{64}\right) - \frac{49}{64}\right] + \frac{3}{2}$

$= 4\left[\left(x + \frac{7}{8}\right)^2 - \frac{49}{64}\right] + \frac{3}{2}$

$= 4\left(x + \frac{7}{8}\right)^2 - \frac{49}{16} + \frac{3}{2} \times 8$

$= 4\left(x + \frac{7}{8}\right)^2 - \frac{49}{16} + \frac{24}{16}$

$= 4\left(x + \frac{7}{8}\right)^2 - \frac{25}{16} = 0$

$\div 4 \quad 4\left(x + \frac{7}{8}\right)^2 = \frac{25}{16} \quad \div 4$

$$= 4\left(x + \frac{7}{8}\right)^2 - \frac{25}{16} = 0$$

$\div 4$ $\quad 4\left(x + \frac{7}{8}\right)^2 = \frac{25}{16}$ $\quad \div 4$

$$\left(x + \frac{7}{8}\right)^2 = \frac{25}{64}$$

$$x + \frac{7}{8} = \pm\sqrt{\frac{25}{64}}$$

$$x + \frac{7}{8} = \pm\frac{5}{8}$$

$$x = -\frac{7}{8} \pm \frac{5}{8}$$

$$x_1 = -\frac{7}{8} + \frac{5}{8} = -\frac{2}{8} = -\frac{1}{4}$$

$$x_2 = -\frac{7}{8} - \frac{5}{8} = -\frac{12}{8} = -\frac{3}{2}$$

$$\left(-\frac{1}{4}, 0\right), \left(-\frac{3}{2}, 0\right)$$

Vertex Shortcut (2.3)

By completing the square with a general quadratic we get a formula to quickly determine the vertex.

$$f(x) = [ax^2 + bx] + c$$

$$= a\left[\left(x^2 + \frac{b}{a}x + k\right) - k\right] + c$$

$$\left(\frac{\text{middle}}{2}\right)^2 = \left(\frac{b}{a} \div \frac{2}{1}\right)^2$$

$$= \left(\frac{b}{a} \times \frac{1}{2}\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$= a\left[\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a^2}\right] + c$$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right] + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

$$a(x-h)^2 + k$$

Vertex: (h, k)

$$h = -\frac{b}{2a} \qquad k = -\frac{b^2}{4a} + c$$

Note: 'a' is the same in standard and general form

Ex 1 Find vertex, x-ints, y-ints and then graph: $f(x) = -3x^2 + 6x + 4$

$ax^2 + bx + c$

$a = -3$

$b = 6$

$c = 4$

$$h = -\frac{b}{2a} = \frac{-(6)}{2(-3)}$$

$$= \frac{-6}{-6} = 1 \quad \leftarrow \text{x value Vertex}$$

① $k = -\frac{b^2}{4a} + c$

$$k = \frac{-(6)^2}{4(-3)} + 4$$

$$= \frac{-36}{-12} + 4$$

$k = 3 + 4 = 7$

② $f(1) = -3(1)^2 + 6(1) + 4$

$= -3 + 6 + 4$

$= 7$

Vertex $(1, 7)$

y-int: $f(0) = -3(0)^2 + 6(0) + 4 = 4$

$(0, 4)$

x-int: use standard form

$f(x) = a(x-h)^2 + k$ $\quad a=-3,\ h=1,\ k=7$

$f(x) = -3(x-1)^2 + 7 = 0$

$-3(x-1)^2 = -7$

$(x-1)^2 = \frac{7}{3}$

$x-1 = \pm\sqrt{\frac{7}{3}}$

$x = 1 \pm \sqrt{\frac{7}{3}}$

For graph

$x_1 = 1+\sqrt{\frac{7}{3}} \approx 2.53$

$x_2 = 1-\sqrt{\frac{7}{3}} \approx -0.53$

$\left(1+\sqrt{\frac{7}{3}}, 0\right), \left(1-\sqrt{\frac{7}{3}}, 0\right)$