The X-Intercept Shortcut (Quadratic Formula)

Friday, November 22, 2019 7:45 AM

3.3/3.4

$f(x) = ax^2 + bx + c$

Vertex $\Rightarrow (h, k)$ $\quad f(x) = a(x-h)^2 + k$

$$h = \frac{-b}{2a} \qquad k = c - \frac{b^2}{4a}$$

x-ints $\Rightarrow f(x) = a(x-h)^2 + k = 0$

$$a(x-h)^2 = -k$$

$$(x-h)^2 = -\frac{k}{a}$$

$$x - h = \pm\sqrt{-\frac{k}{a}}$$

$$x = (h) \pm \sqrt{-\frac{(k)}{a}}$$

$$x = \left(\frac{-b}{2a}\right) \pm \sqrt{-\left(\frac{\left(c - \frac{b^2}{4a}\right)}{a}\right)}$$

$$= \frac{-b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a} \cdot 4a}$$

$$= \frac{-b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{4ac}{4a^2}}$$

$$= \frac{-b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$= \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

2, 1 or 0 Solutions

$$\frac{b^2}{4a} \div a = \frac{b^2}{4a} \times \frac{1}{a} = \frac{b^2}{4a^2}$$

$$\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$$

Ex1 Solve using quadratic formula (then check answers)

a) $5x^2 - 2x + 10 = 0$ (ax^2+bx+c)

$a = 5$, $b = -2$, $c = 10$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(10)}}{2(5)}$$

$$= \frac{2 \pm \sqrt{4 - 200}}{10}$$

$$= \frac{2 \pm \sqrt{-196}}{10}$$

neg root

no soln (solution)

b) $\left(\sqrt{x+12}\right)^2 = (3-2x)^2$

$$x + 12 = (3-2x)(3-2x)$$

$$x + 12 = 9 - 6x - 6x + 4x^2$$

$$0 = 4x^2 - 13x - 3$$

$a = 4$, $b = -13$, $c = -3$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(4)(-3)}}{2(4)}$$

$$x = \frac{13 \pm \sqrt{169+48}}{8} = \frac{13 \pm \sqrt{217}}{8}$$

Check use decimal $x \approx 3.466, -0.216$

$x \approx 3.466$

$\sqrt{3.466+12} = 3 - 2(3.466)$

$3.93268\ldots = -3.932$

$x \approx -0.216$

$\sqrt{-0.216+12} = 3 - 2(-0.216)$

$3.43278\ldots = 3.432$

Close (rounding) enough

Answer: $x = \frac{13-\sqrt{217}}{8}$

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c) $-2x^4 + 3x^2 + 5 = 0$

$z = x^2$

$-2z^2 + 3z + 5 = 0$

$a = -2$
$b = 3$
$c = 5$

$z = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$z = \frac{-3 \pm \sqrt{(3)^2-4(-2)(5)}}{2(-2)}$

$z = \frac{-3 \pm \sqrt{9+40}}{-4}$

$$z = \frac{-3 \pm \sqrt{9 + 40}}{2(-2)}$$

$$z = \frac{-3 \pm \sqrt{49}}{-4} = \frac{-3 \pm 7}{-4}$$

$$z = \frac{-3+7}{-4} = \frac{4}{-4} = -1$$

$$z = \frac{-3-7}{-4} = \frac{-10}{-4} = \frac{5}{2}$$

$z = x^2$

$\Rightarrow x^2 = -1$ (or) $x^2 = \frac{5}{2}$

$x = \pm\sqrt{-1}$ — No Soln

$x = \pm\sqrt{\frac{5}{2}}$ — 2 Soln

Check $x \approx \pm 1.581\ldots$

$x \approx 1.581\ldots$

$-2(1.581)^4 + 3(1.581)^2 + 5 = 0$

$0.00307\ldots = 0$ close enough (rounded)

$x \approx -1.581\ldots$

$-2(-1.581)^4 + 3(1.581)^2 + 5 = 0$

$0.00307\ldots = 0$

$x = \pm\sqrt{\frac{5}{2}}$

(3.4) Discriminant

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

0 Soln: $b^2 - 4ac < 0$

2 Soln: $b^2 - 4ac > 0$

1 Soln: $b^2 - 4ac = 0$

↳ $x = \frac{-b \pm \sqrt{0}}{2a} = \frac{-b \pm 0}{2a} = \frac{-b}{2a}$

$D = b^2 - 4ac \Rightarrow$ Discriminant

Ex 2 How many Solutions exist for:

a) $-14x^2 + 8x - 10 = 0$

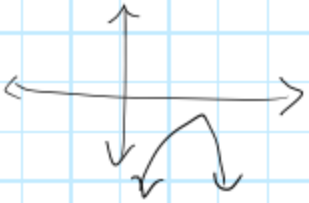
$D = b^2 - 4ac$

$= 8^2 - 4(-14)(-10)$

$= 64 - 560$

$= -496$

$\Rightarrow$ NO Soln



b) $-2x^2 + 5x + 14 = 0$

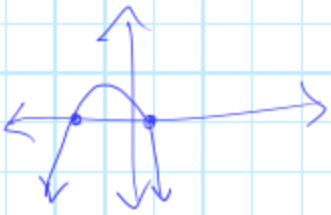
$D = b^2 - 4ac$

$= 5^2 - 4(-2)(14)$

$= 25 + 112$

$= 137$

$\Rightarrow$ 2 Soln

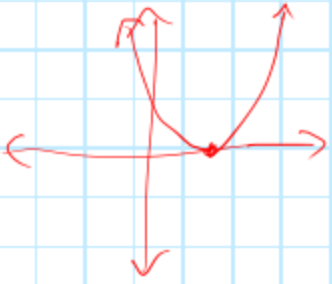


c) $8x^2 - 40x + 50 = 0$

$D = (-40)^2 - 4(8)(50)$

$= 1600 - 1600$

$= 0 \Rightarrow$ 1 Soln



Ex3 Find the values of p so that $3x^2 + px + 12 = 0$ has:

a) 1 Sol^n b) No Sol^n c) 2 Sol^n

$D = b^2 - 4ac$

$= p^2 - 4(3)(12)$

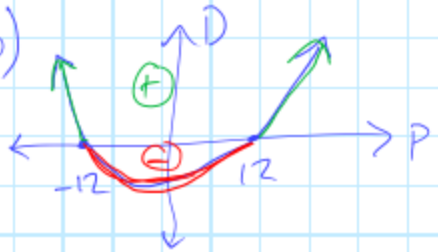
$D = p^2 - 144$

a) $p^2 - 144 = 0$

$(p-12)(p+12) = 0$

$p = \pm 12$

b)



b) No Sol^n

$D < 0$

$-12 < p < 12$

c) 2 Sol^n

$D > 0$

$p < -12, \quad p > 12$