The X-Intercept Shortcut (Quadratic Formula)

Friday, November 22, 2019 7:45 AM

3.3/3.4

$f(x) = ax^2 + bx + c$

vertex $\Rightarrow$ $f(x) = a(x-h)^2 + k$

vertex (h, k) where $\boxed{h = \frac{-b}{2a}}$

$\boxed{k = c - \frac{b^2}{4a}}$

x-int $\Rightarrow$ $f(x) = a(x-h)^2 + k = 0$

$$a(x-h)^2 = -k$$

$$(x-h)^2 = \frac{-k}{a}$$

$$x - h = \pm\sqrt{-\frac{k}{a}}$$

$$x = h \pm \sqrt{-\frac{k}{a}}$$

$$x = -\frac{b}{2a} \pm \sqrt{-\left(\frac{c - \frac{b^2}{4a}}{a}\right)}$$

$$= -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c \cdot 4a}{a \cdot 4a}}$$

$$= -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{4ac}{4a^2}}$$

$$= -\frac{b}{2a} \pm \sqrt{\frac{b^2-4ac}{4a^2}}$$

$$\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$$

$$= -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}}$$

$$= -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \boxed{\frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

0, 1, or 2 solutions

Ex 1 Solve using quadratic formula (then check answers)

a) $5x^2 - 2x + 10 = 0$

$a = 5$
$b = -2$
$c = 10$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(10)}}{2(5)}$$

$$= \frac{2 \pm \sqrt{4 - 200}}{10}$$

$$= \frac{2 \pm \sqrt{-196}}{10} \leftarrow \text{neg root}$$

NO Sol^{n}s

b) $\left(\sqrt{x+12}\right)^2 = (3-2x)^2$ (Need to check)

$$x + 12 = (3-2x)(3-2x)$$

$$x + 12 = 9 - 6x - 6x + 4x^2$$

$$\begin{array}{l} -x \quad -12 \\ x + 12 = 9 - 12x + 4x^2 \quad -x - 12 \end{array}$$

$$0 = 4x^2 - 13x - 3$$

$a = 4$
$b = -13$
$c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(4)(-3)}}{2(4)}$$

$$x = \frac{13 \pm \sqrt{169 + 48}}{8}$$

$$x = \frac{\left(13 \pm \sqrt{217}\right)}{8}$$

$x \approx 3.466, -0.216$

Check $x \approx 3.466$ $\overset{?}{\approx}$

$\sqrt{(3.466)+12} = 3-2(3.466)$

$3.93 \cancel{=} -3.932$

$x \approx -0.216$

$\sqrt{-0.216+12} = 3-2(-0.216)$

$3.433 = 3.432$ ✓ $\overset{?}{=}$

$$\boxed{x = \frac{13-\sqrt{217}}{8}}$$

- Any time the starting equation has weird stuff ($\sqrt{x+12}$ or $\frac{5}{x-3}$ or x^4) you might end up with extra answers that don't work
- Regular quadratics don't give extra answers

c) $-2x^4 + 3x^2 + 5 = 0$

$z = x^2$

$-2z^2 + 3z + 5 = 0$

$a = -2$
$b = 3$
$c = 5$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-3 \pm \sqrt{(3)^2 - 4(-2)(5)}}{2(-2)}$$

$$z = \frac{-3 \pm \sqrt{9 + 40}}{-4}$$

$$z = \frac{-3 \pm \sqrt{49}}{-4} = \frac{-3 \pm 7}{-4}$$

$$z = \frac{4}{-4} = \boxed{-1}, \quad = \frac{-10}{-4} = \boxed{\frac{5}{2}}$$

$Z = x^2$

$\Rightarrow x^2 = -1$ (or) $x^2 = \frac{5}{2}$

$x = \pm\sqrt{-1}$ $\quad$ $x = \pm\sqrt{\frac{5}{2}} \approx \pm 1.581$

no answers

Check $x = 1.581$

$-2(1.581)^4 + 3(1.581)^2 + 5 = 0$

$0.00307 = 0$ ✓ close enough

$x = -1.581$

$-2(-1.581)^4 + 3(1.581)^2 + 5 = 0$

$0.00307 = 0$

ok. ✓

Discriminant: (3.4)

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

0 solutions: $b^2-4ac < 0$

2 Solutions: $b^2-4ac > 0$

1 Solution: $b^2-4ac = 0$

$\hookrightarrow x = \frac{-b \pm \sqrt{0}}{2a} = \frac{-b \pm 0}{2a} = \frac{-b}{2a}$

$b^2-4ac = D$ (Discriminant)

Ex 2 How many roots exist for the following equations?

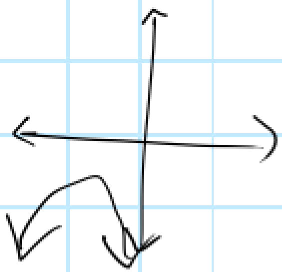
a) $-14x^2+8x-10=0$

$D=b^2-4ac$

$=8^2-4(-14)(-10)$

$=64-560$

$=-496 \Rightarrow$ NO $\text{sol}^{\underline{n}}$



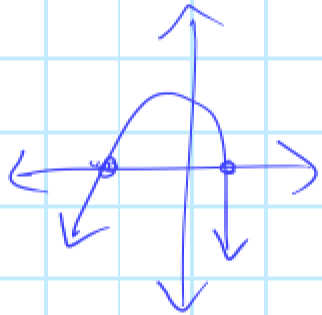
b) $-2x^2+5x+14=0$

$D=5^2-4(-2)(14)$

$=25+112$

$=137$

$\Rightarrow 2\ \text{sol}^{\underline{n}}$



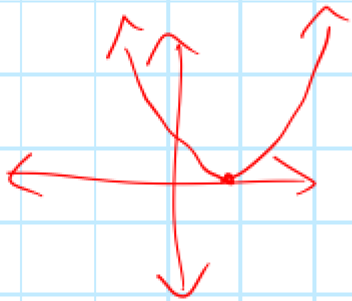
c) $8x^2-40x+50=0$

$D=(-40)^2-4(8)(50)$

$=1600-1600$

$=0$

$\Rightarrow 1\ \text{sol}^{\underline{n}}$



Ex 3 Find the values of P so that

$3x^2+px+12$ has:

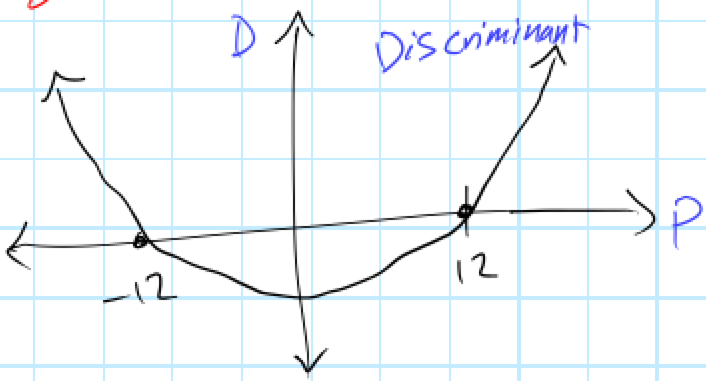
a) 1 Sol$^{\underline{n}}$ b) 0 Sol$^{\underline{n}}$ c) 2 Sol$^{\underline{n}}$

$$D = b^2 - 4ac$$

$$= p^2 - 4(3)(12)$$

$$D = p^2 - 144$$

$$\overset{y}{D} = (\overset{x}{p}+12)(\overset{x}{p}-12)$$



a) 1 Sol$^{\underline{n}}$

$\overset{y}{D} = 0$

$\underline{p = 12, -12}$

b) 0 Sol$^{\underline{n}}$

$D < 0$

$-12 < P < 12$

c) 2 Sol$^{\underline{n}}$

$D > 0$

$P < -12, \; P > 12$