The X-Intercept Shortcut (Quadratic Formula)

Friday, November 22, 2019 7:45 AM

$f(x) = ax^2 + bx + c$

Vertex $\Rightarrow (h, k)$ $\quad f(x) = a(x-h)^2 + k$

$$\boxed{h = -\frac{b}{2a} \quad | \quad k = c - \frac{b^2}{4a}}$$

X-int $\Rightarrow$ $f(x) = a(x-h)^2 + k = 0$

$$a(x-h)^2 = -k$$

$$(x-h)^2 = -\frac{k}{a}$$

$$x - h = \pm\sqrt{-\frac{k}{a}}$$

$$x = (h) \pm \sqrt{-\frac{(k)}{a}}$$

$$x = \left(\frac{-b}{2a}\right) \pm \sqrt{-\left(\frac{\left(c - \frac{b^2}{4a}\right)}{a}\right)}$$

$$= -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a} \cdot 4a}$$

$$= -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{4ac}{4a^2}}$$

$$= -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$= -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \boxed{\frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

2, 1, or 0 sol^{n}s (solutions)

Ex 1 Solve using quadratic formula (then check answers)

a) $5x^2 - 2x + 10 = 0$

$a = 5$

$b = -2$

$c = 10$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(10)}}{2(5)}$$

$$x = \frac{2 \pm \sqrt{4 - 200}}{10}$$

$$x = \frac{2 \pm \sqrt{-196}}{10}$$ ← neg root

No Sol^{n}s

b) $\left(\sqrt{x+12}\right)^2 = (3-2x)^2$

$$x + 12 = (3-2x)(3-2x)$$

$$\overset{-x}{x} \overset{-12}{+12} = 9 - 6x - 6x + 4x^2$$

$$0 = 4x^2 - 13x - 3$$

$a = 4$

$b = -13$

$c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(4)(-3)}}{2(4)}$$

$$x = \frac{13 \pm \sqrt{169 + 48}}{8}$$

$$x = \left(\frac{13 \pm \sqrt{217}}{8}\right)$$

Check use decimals/calc $x \approx 3.466, -0.216$

Check $x \approx 3.466$

$\sqrt{3.466 + 12} = 3 - 2(3.466)$

$3.93268 = -3.932$

$x \approx -0.216$

$\sqrt{-0.216 + 12} = 3 - 2(-0.216)$

$3.43278 = 3.432$ ✓ close enough !!

$$x = \frac{13 - \sqrt{217}}{8}$$

- If the question has weird stuff ($\sqrt{x+12}$ or $\frac{5}{x-3}$ or x^4) we might end up with extra answers that don't work

- Regular quadratics don't give extra answers

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c) $-2x^4 + 3x^2 + 5 = 0$

$x^2 = z$

$-2z^2 + 3z + 5 = 0$

$a = -2$

$b = 3$

$c = 5$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-3 \pm \sqrt{(3)^2 - 4(-2)(5)}}{2(-2)}$$

$$= \frac{-3 \pm \sqrt{9 + 40}}{-4}$$

$$= \frac{-3 \pm \sqrt{49}}{-4} = \frac{-3 \pm 7}{-4}$$

$z = \frac{4}{-4} = |-1|$ or $= \frac{-10}{-4} = \left|\frac{5}{2}\right|$

$z = x^2$

$\Rightarrow x^2 = -1$ (or) $x^2 = \frac{5}{2}$

$x = \pm\sqrt{-1}$ $\qquad$ $x = \pm\sqrt{\frac{5}{2}}$

No Sol^{n}

$x \approx \pm 1.581$

check $x \approx 1.581$

$-2(1.581)^4 + 3(1.581)^2 + 5 = 0$

$0.0030726 = 0$ ✓

close enough (rounding)

$x \approx -1.581$

$-2(-1.581)^4 + 3(-1.581)^2 + 5 = 0$

$0.0030726 = 0$ ✓

$x = \pm\sqrt{\frac{5}{2}}$

Discriminant:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

0 Sol^n : $b^2 - 4ac < 0$

2 Sol^n : $b^2 - 4ac > 0$

1 Sol^n : $b^2 - 4ac = 0$

↳ $x = \frac{-b \pm \sqrt{0}}{2a} = \frac{-b \pm 0}{2a} = \frac{-b}{2a}$

$D = b^2 - 4ac$ ← Discriminant

Ex2 How many solutions exist for:

a) $-14x^2 + 8x - 10 = 0$

$D = b^2 - 4ac$

b) $-2x^2 + 5x + 14 = 0$

$D = 5^2 - 4(-2)(14)$

c) $8x^2 - 40x + 50 = 0$

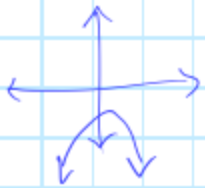
$D = (-40)^2 - 4(8)(50)$

$D = b^2 - 4ac$

$= 8^2 - 4(-14)(-10)$

$= 64 - 560$

$= -496 \Rightarrow$ NO Sol^n



$D = 5^2 - 4(-2)(14)$

$= 25 + 112$

$= 137$

$\Rightarrow 2$ Sol^n

$D = (-40)^2 - 4(8)(50)$

$= 1600 - 1600$

$= 0 \Rightarrow 1$ Sol^n

Ex 3 Find the values of p so that

$3x^2 + px + 12$ has

a) 1 Sol^n b) No Sol^n c) 2 Sol^n

$D = b^2 - 4ac$

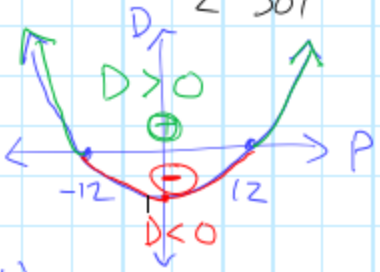
$= p^2 - 4(3)(12)$

$D = 1p^2 - 144$

$D = (p-12)(p+12)$

$(p-12)(p+12) = 0$

a) $p = \pm 12$



b) No Sol^n

$D < 0$

$\Rightarrow -12 < p < 12$

c) 2 Sol^n $D > 0$

$p < -12, \quad p > 12$