Introduction to Radicals

Tuesday, February 11, 2020

5.1/5.2

Radical $\Leftrightarrow$ Root

one thing

index — radical

Definition

$x^n = a \Rightarrow x = \sqrt[n]{a}$ ← radicand

E.g. $x^2 = 4 \Rightarrow x = \pm\sqrt[2]{4} = \pm 2$

$x^3 = 8 \Rightarrow x = \sqrt[3]{8} = 2$

radicand is...

index is...	+	−
even	$x^2 = 4 \Rightarrow x = \pm\sqrt{4} = \pm 2$ $x^4 = 81 \Rightarrow x = \pm\sqrt[4]{81} = \pm 3$ 2 answers Check: $(3)^4 = 81$ $(-3)^4 = 81$	$x^2 = -4 \Rightarrow x = \pm\sqrt{-4} = \emptyset$ $x^4 = -81 \Rightarrow x = \pm\sqrt[4]{-81} = \emptyset$ No answers even power result always +
odd	$x^3 = 27 \Rightarrow x = \sqrt[3]{27} = 3$ $x^5 = 32 \Rightarrow x = \sqrt[5]{32} = 2$ 1 answer Check: $(2)^5 = 32$ BUT $\cancel{(-2)^5 = 32}$	$x^3 = -27 \Rightarrow x = \sqrt[3]{-27} = -3$ $x^5 = -32 \Rightarrow x = \sqrt[5]{-32} = -2$ 1 answer

NB: ← Nota Bene (take note)

If the radicand is zero, only 1 answer $x = 0$

$x^2 = 0 \Rightarrow x = 0$

$x^5 = 0 \Rightarrow x = 0$

even or odd index

① $x^2 = 9 \Rightarrow x = \pm\sqrt{9} = \pm 3$ ← 2 answers

BUT ② $\sqrt{9} = 3$ ← 1 answer

Calculators $\sqrt[5]{243} = \boxed{5}\ \boxed{\sqrt[x]{\ }}\ \boxed{243} = 3$

Only baby calculators allowed for this unit

Properties of Radicals

① $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

$\sqrt{(-2)\cdot(-2)} = \sqrt{4} = 2$

②a $\sqrt[n]{a\cdot b} = \sqrt[n]{a}\cdot\sqrt[n]{b}$ $\quad$ ②a $\rightarrow \cancel{\sqrt{-2}\cdot\sqrt{-2} = \emptyset}$

$a, b \geq 0$

②b $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

③ $\sqrt[n]{a^n} = a$ ← True, most of the time $(a \geq 0)$

$\sqrt[n]{a^n} \stackrel{?}{=} a$	$a > 0$	$a < 0$
n is even	$\sqrt[2]{3^2} = \sqrt[2]{9} = 3$ ✓ $\sqrt[4]{2^4} = \sqrt[4]{16} = 2$	$\sqrt[2]{(-3)^2} = \sqrt[2]{9} = 3$ $\sqrt[4]{(-2)^4} = \sqrt[4]{16} = 2$ ×
n is odd	$\sqrt[3]{3^3} = \sqrt[3]{27} = 3$ ✓	$\sqrt[3]{(-3)^3} = \sqrt[3]{-27} = -3$ ✓

If n is odd, $\sqrt[n]{a^n} = a$

If n is even, $\sqrt[n]{a^n} = |a|$ ← always (+) ← absolute value e.g. $|5| = 5$ $\quad -|-5| = -5$

$|-5| = 5$

f(x)

|f(x)|

Mixed/Entire Radicals

we will be using $\underline{\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}}$

Simplify: take out as much as possible from the root

$2x = 2 \cdot x$

Simplify $\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2 \cdot \sqrt{5}$

Perf sq

other stuff

fast method

$= 2\sqrt{5}$

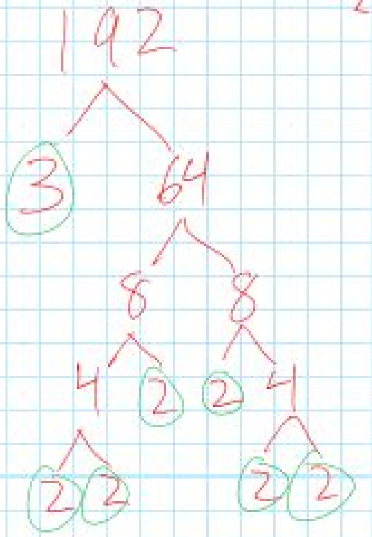
Method

$$\sqrt[3]{192} = \sqrt[3]{3\cdot\underbrace{(2\cdot 2\cdot 2)}_{2^3}\underbrace{(2\cdot 2\cdot 2)}_{2^3}} = \sqrt[3]{3}\cdot\sqrt[3]{2^3}\cdot\sqrt[3]{2^3}$$

$$= \sqrt[3]{3}\cdot 2\cdot 2$$

$$= \underline{4\sqrt[3]{3}}$$

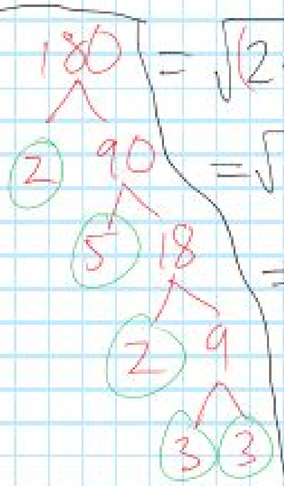
For big numbers, we use factor tree



Prime numbers

$$\sqrt{180x^6y^5} = \sqrt{180}\cdot\sqrt{x^6}\cdot\sqrt{y^5}$$

$\boxed{x, y \geq 0}$



$$= \sqrt{(2\cdot 2)\cdot 5\cdot(3\cdot 3)}\cdot\sqrt{x^2\cdot x^2\cdot x^2}\cdot\sqrt{y^2\cdot y^2\cdot y}$$

$$= \sqrt{2^2}\cdot\sqrt{5}\cdot\sqrt{3^2}\cdot\sqrt{x^2}\cdot\sqrt{x^2}\cdot\sqrt{x^2}\cdot\sqrt{y^2}\cdot\sqrt{y^2}\cdot\sqrt{y}$$

$$= 2\cdot\sqrt{5}\cdot 3\cdot x\cdot x\cdot x\cdot y\cdot y\cdot\sqrt{y}$$

$$= \underline{6x^3y^2\sqrt{5y}}$$

Backwards (mixed to entire)

$$4x^2\sqrt{6x} = \sqrt{(4x^2)^2\,6x} = \sqrt{16x^4\cdot 6x} = \sqrt{96x^5}$$