Introduction to Radicals

Tuesday, February 11, 2020 8:40 AM

Radicals $\Leftrightarrow$ Roots

Single 'thing'

Definition

index

radical

$$x^n = a \Rightarrow x = \sqrt[n]{a} = a^{\frac{1}{n}}$$

radicand

E.g.

$$x^3 = 8 \Rightarrow x = \sqrt[3]{8} = 2$$

$$x^2 = 9 \Rightarrow x = \pm\sqrt[2]{9} = \pm 3$$

Radicand is...

index is...	$+$	$-$
even	$x^2 = 9 \Rightarrow x = \pm\sqrt{9} = \pm 3$ $x^4 = 16 \Rightarrow x = \pm\sqrt[4]{16} = \pm 2$ $\boxed{\text{2 answers}}$ check: $(2)^4 = 16$ $(-2)^4 = 16$	$x^2 = -9 \Rightarrow x = \pm\sqrt{-9} = \emptyset$ $x^4 = -16 \Rightarrow x = \pm\sqrt[4]{-16} = \emptyset$ always $+$ $\boxed{\text{No answer}}$
odd	$x^3 = 27 \Rightarrow x = \sqrt[3]{27} = 3$ $x^5 = 32 \Rightarrow x = \sqrt[5]{32} = 2$ $\boxed{\text{1 answer}}$ check: $(2)^5 = 32$	$x^3 = -27 \Rightarrow x = \sqrt[3]{-27} = -3$ $x^5 = -32 \Rightarrow x = \sqrt[5]{-32} = -2$ $\boxed{\text{1 answer}}$ $(-2)^5 = -32$

BUT $\cancel{(-2)^5 = -32}$

on calc $\boxed{5}\ \boxed{\sqrt[x]{\ }}\ \boxed{-32}$

For this unit ONLY

baby calc

NB: ← Nota Bene (take note)

If radicand is zero, only $\underline{1 \text{ answer}}$ $(x=0)$

$x^2 = 0 \Rightarrow x = 0$

$x^5 = 0 \Rightarrow x = 0$

even or odd $x=0$

① $x^2 = 9 \Rightarrow x = \pm\sqrt[2]{9} = \underline{\pm 3}$ ← 2 answers

However: ② $\sqrt{9} = \underline{3}$ ← 1 answer

Radical Properties

① $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

②a $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

$\boxed{a, b \geq 0}$

$\sqrt{(-2)\cdot(-2)} = \sqrt{4} = \underline{2}$

$= \sqrt{-2} \cdot \sqrt{-2}$

$= \phi$

②b $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

③ $\sqrt[n]{a^n} = a$ ← True, most of the time...

$\boxed{a > 0}$

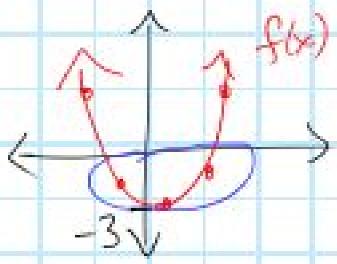
$\sqrt[n]{a^n} \stackrel{?}{=} a$	$a>0$	$a<0$
n is even	$\sqrt{5^2}=\sqrt{25}=5$ ✓ $\sqrt[4]{2^4}=\sqrt[4]{16}=2$ ✓	$\sqrt{(-5)^2}=\sqrt{25}=5$ $\sqrt[4]{(-2)^4}=\sqrt[4]{16}=2$
n is odd	$\sqrt[3]{3^3}=\sqrt[3]{27}=3$ ✓	$\sqrt[3]{(-3)^3}=\sqrt[3]{-27}=-3$

If n is odd, $\sqrt[n]{a^n} = a$

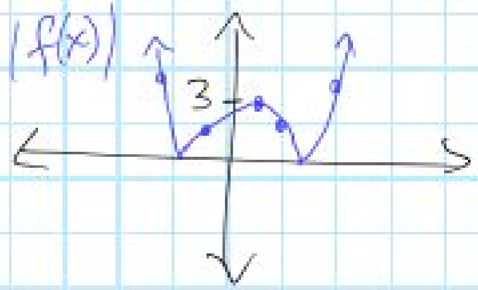
If n is even, $\sqrt[n]{a^n} = |a|$ always (+) absolute value

E.g. $|5| = 5$ $\quad -|-5| = -5$

$|-5| = 5$



Graph original, then flip (−)



Mixed/ Entire Radicals

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Simplify: remove as much as you can from the root

$2 \cdot x = 2x$

NOT $\sqrt[2]{5}$

Simplify $\sqrt{20} = \sqrt{4} \cdot \sqrt{5} = 2 \cdot \sqrt{5} = 2\sqrt{5}$

perfect sq

other stuff

fast

$$\sqrt[3]{192} = \sqrt[3]{\underbrace{(2 \cdot 2 \cdot 2)}_{2^3} \cdot \underbrace{(2 \cdot 2 \cdot 2)}_{2^3} \cdot 3} = \sqrt[3]{2^3} \cdot \sqrt[3]{2^3} \cdot \sqrt[3]{3}$$

$$= 2 \cdot 2 \cdot \sqrt[3]{3}$$

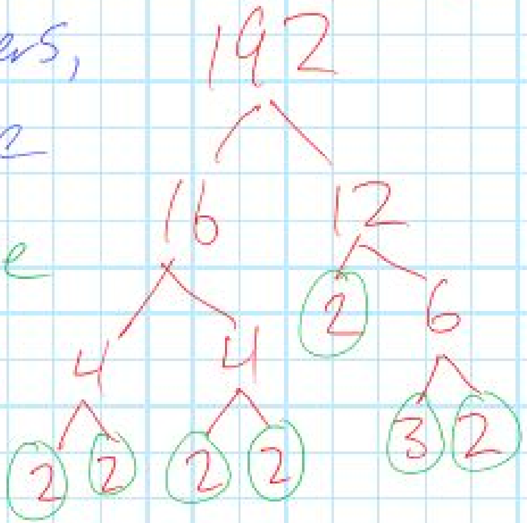
$$= 4\sqrt[3]{3}$$

$\sqrt[3]{2^6} = \sqrt[3]{(2^2)^3}$

$2^2 = 4$

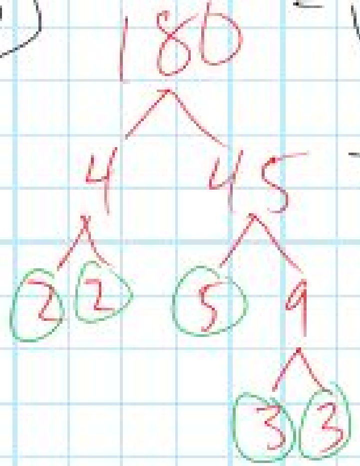
For big numbers, prime factor tree

prime



$$\sqrt{180x^6y^5} = \sqrt{180}\cdot\sqrt{x^6}\cdot\sqrt{y^5}$$

$\boxed{x, y \geq 0}$



$$= \sqrt{(2\cdot 2)\cdot 5\cdot (3\cdot 3)}\cdot\sqrt{x^2\cdot x^2\cdot x^2}\cdot\sqrt{y^2\cdot y^2\cdot y}$$

$$= \sqrt{2^2}\cdot\sqrt{5}\cdot\sqrt{3^2}\cdot\sqrt{x^2}\cdot\sqrt{x^2}\cdot\sqrt{x^2}\cdot\sqrt{y^2}\cdot\sqrt{y^2}\cdot\sqrt{y}$$

$$= 2\cdot\sqrt{5}\cdot 3\cdot x\cdot x\cdot x\cdot y\cdot y\cdot\sqrt{y}$$

$$= \underline{6x^3y^2\sqrt{5y}}$$

<u>Backwards</u> (Mixed to entire)

$$4x^2\sqrt{6x} = \sqrt{(4x^2)^2\cdot 6x} = \sqrt{16x^4\cdot 6x} = \sqrt{96x^5}$$