Introduction to Radicals

Tuesday, February 11, 2020 8:40 AM

Radical $\Leftrightarrow$ Root

Definition

$$x^n = a \Leftrightarrow x = \sqrt[n]{a}$$

index

radical

radicand

e.g. $x^2 = 4 \Rightarrow x = \pm\sqrt{4}$

$x^3 = 8 \Rightarrow x = \sqrt[3]{8}$

Radicand is...

index is...	$+$	$-$
even	$x^2 = 4 \Rightarrow x = \pm\sqrt[2]{4} = \pm 2$ $x^4 = 81 \Rightarrow x = \pm\sqrt[4]{81} = \pm 3$ $\boxed{\text{2 answers}}$ check: $(3)^4 = 81$ and $(-3)^4 = 81$	$x^2 = -4 \Rightarrow x = \sqrt[2]{-4} = \varnothing$ (no answer) $x^4 = -81 \Rightarrow x = \sqrt[4]{-81}$ $\boxed{\text{no answer}}$ even powers always give $+$ results
odd	$x^3 = 8 \Rightarrow x = \sqrt[3]{8} = 2$ $x^5 = 32 \Rightarrow x = \sqrt[5]{32} = 2$ 1 answer check: $(2)^5 = 32$ but $\cancel{(-2)^5 = 32}$	$x^3 = -8 \Rightarrow x = \sqrt[3]{-8} = -2$ $x^5 = -32 \Rightarrow x = \sqrt[5]{-32} = -2$ 1 answer

N.B.: ← Nota Bene (take note)

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If radicand is zero, only 1 answer

$x^2 = 0 \Rightarrow x = 0$

$x^5 = 0 \Rightarrow x = 0$

(even or odd index)

① $x^2 = 9$ has 2 answers $(\pm\sqrt{9} = \pm 3)$

BUT ② $\sqrt{9} = 3$ only (not $\pm$)

Radical properties

① $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

② $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$(a, b \geq 0)$

Be careful with $\ominus$

$\sqrt[2]{-2 \cdot -2} = \sqrt[2]{4} = 2$

$\cancel{= \sqrt[2]{-2} \cdot \sqrt[2]{-2}}$

③ $\sqrt[n]{a^n} = \left(\sqrt[n]{a}\right)^n = a$ ← mostly correct, but not always...

$(a \geq 0)$

e.g. $(\sqrt{4})^{\cancel{2}} = (2)^2 = 4$

$\hookrightarrow = 4$

$(\sqrt[\cancel{3}]{27})^{\cancel{3}} = (3)^3 = 27$

$\hookrightarrow 27$

$(\sqrt{-4})^2 = \emptyset$ } don't match

$\hookrightarrow = -4$

$(\sqrt[\cancel{3}]{-27})^{\cancel{3}} = (-3)^3 = -27$

$\hookrightarrow = -27$

$\boxed{\sqrt[n]{a^n} \stackrel{?}{=} a}$

	$a > 0$	$a < 0$
n is even	$\sqrt[2]{3^{\cancel{2}}} = \sqrt[2]{9} = 3$ $\hookrightarrow = 3$ ✓	$\sqrt[2]{(-3)^2} = \sqrt[2]{9} = 3$ $\hookrightarrow \cancel{=} -3$
n is odd	$\sqrt[\cancel{5}]{2^{\cancel{5}}} = \sqrt[5]{32} = 2$ $\hookrightarrow = 2$ ✓	$\sqrt[5]{(-2)^5} = \sqrt[5]{-32} = -2$ $\hookrightarrow = -2$ ✓

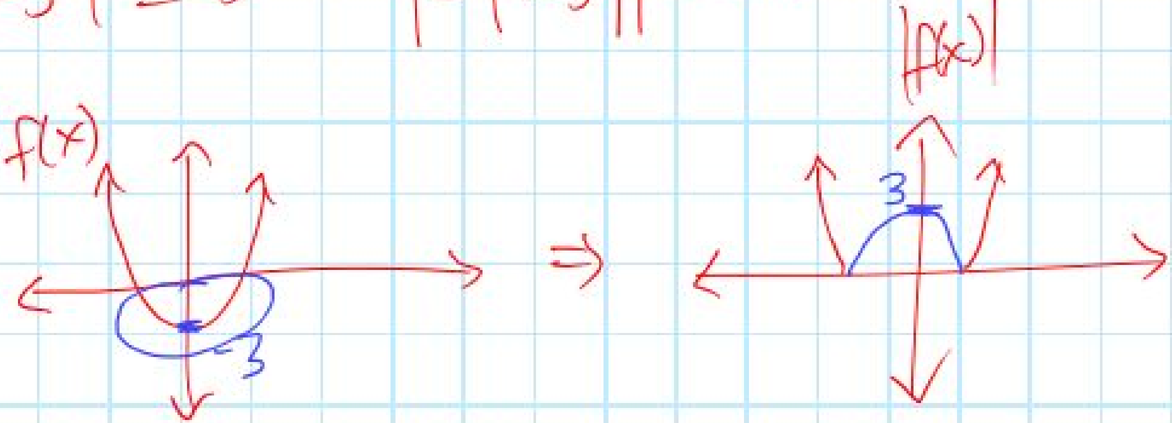
For n is odd $\Rightarrow$ $\sqrt[n]{a^n} = a$ everytime

For n is even $\Rightarrow$ $\sqrt[n]{a^n} = |a|$ ← always (+)

absolute value

Eg. $|5| = 5$ $\quad -|-5| = -5$

$|-5| = 5$ $\quad ||-|-5|| = 5$



Mixed to Entire Radicals

For this we will be using $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Simplify/Reduce - remove as much as possible from the root

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not $3\sqrt{5}$

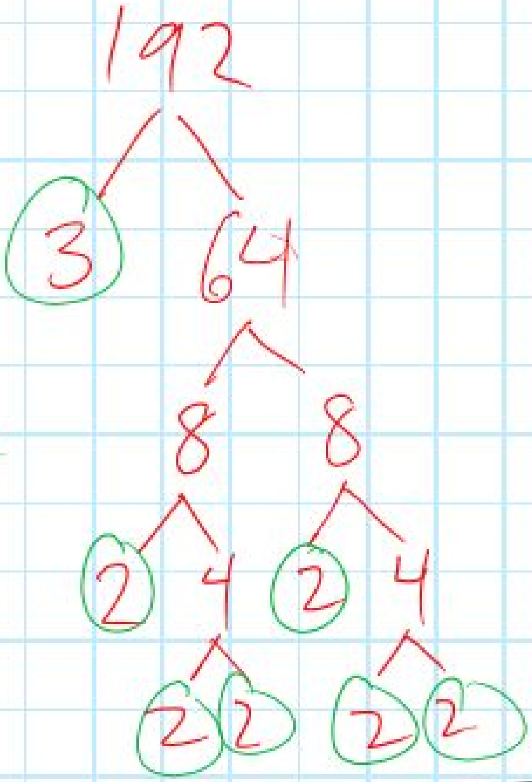
simplify $\sqrt{20} = \sqrt{\underset{\substack{\uparrow \\ \text{perfect squares}}}{4} \cdot \underset{\nwarrow \text{ other stuff}}{5}} = \sqrt{4} \cdot \sqrt{5} = 2 \cdot \sqrt{5} = 2\sqrt{5}$

$$\sqrt[3]{192} = \sqrt[3]{3 \cdot (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)} = \sqrt[3]{3} \cdot \sqrt[3]{2^3} \cdot \sqrt[3]{2^3}$$

$$= \sqrt[3]{3} \cdot 2 \cdot 2$$

$$= \underline{4\sqrt[3]{3}}$$

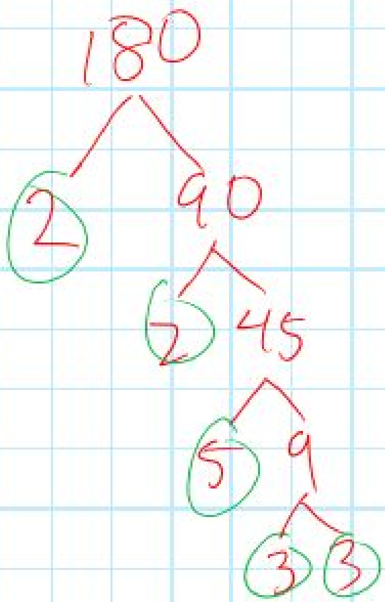
For big numbers, want to use factor tree



Find the ends

$$\sqrt{180x^6y^5} = \sqrt{180}\cdot\sqrt{x^6}\cdot\sqrt{y^5}$$

$\boxed{x, y \ge 0}$



$$= \sqrt{2\cdot 2\cdot 5\cdot 3\cdot 3}\cdot\sqrt{x^2\cdot x^2\cdot x^2}\cdot\sqrt{y^2\cdot y^2\cdot y}$$

$$= \sqrt{2^2}\cdot\sqrt{5}\cdot\sqrt{3^2}\cdot\sqrt{x^2}\cdot\sqrt{x^2}\cdot\sqrt{x^2}\cdot\sqrt{y^2}\cdot\sqrt{y^2}\cdot\sqrt{y}$$

$$= 2\cdot\sqrt{5}\cdot 3\cdot x\cdot x\cdot x\cdot y\cdot y\cdot\sqrt{y}$$

$$= 6x^3y^2\sqrt{5y}$$

Backwards (mixed to entire)

$$4x^2\sqrt{6x} = \sqrt{(4x^2)^2\,6x} = \sqrt{16\cdot x^4\cdot 6x} = \sqrt{96x^5}$$