

1. Given the parabola: $f(x) = \frac{1}{4}x^2 + 3x - 10$

- a. Find the axis of symmetry of $f(x)$ by completing the square (no shortcut allowed!)
 b. Find the x-intercepts of $f(x)$ (give exact answers)

$$\begin{aligned} a) & \frac{1}{4} \left[(x^2 + 12x + k) - k \right] - 10 \\ & = \frac{1}{4} \left[(x^2 + 12x + 36) - 36 \right] - 10 \\ & = \frac{1}{4} \left[(x+6)^2 - 36 \right] - 10 = \frac{1}{4} (x+6)^2 - 9 - 10 \\ & = \frac{1}{4} (x+6)^2 - 19 \\ & h = -6 \quad \boxed{x = -6} \end{aligned}$$

$$b) \frac{1}{4} (x+6)^2 - 19 = 0$$

$$\frac{1}{4} (x+6)^2 = 19$$

$$(x+6)^2 = 76$$

$$x+6 = \pm \sqrt{76}$$

$$x = -6 \pm \sqrt{76}$$

$$\boxed{(-6 \pm \sqrt{76}, 0)}$$

2. Given the parabola: $f(x) = 3x^2 + 6x - 4$

- a. Find the vertex of $f(x)$ (shortcut is allowed)
 b. Find the x-intercepts and y-intercept of $f(x)$ (give exact answers)
 c. Sketch $f(x)$ on the grid provided, plotting all intercepts and vertex



a. vertex: $(-1, -7)$

b. y-int: $(0, -4)$

x-int(s): $(-1 \pm \sqrt{\frac{7}{3}}, 0)$

$$a) h = \frac{-b}{2a} = \frac{-6}{2(3)} = -1$$

$$\begin{aligned} f(-1) &= k = 3(-1)^2 + 6(-1) - 4 \\ &= 3 - 6 - 4 = -7 \end{aligned}$$

$$b) f(x) = 3(x+1)^2 - 7 = 0$$

$$3(x+1)^2 = 7$$

$$(x+1)^2 = \frac{7}{3}$$

$$x+1 = \pm \sqrt{\frac{7}{3}}$$

$$x = -1 \pm \sqrt{\frac{7}{3}}$$

$$\approx 0.5, -2.5$$

$$f(0) = 3(0+1)^2 - 7$$

$$= 3(1)^2 - 7$$

$$= 3 - 7$$

$$= -4$$

3. Mr. Johnston is deciding on a price for Byng Wear sweaters. He is currently selling them for \$52 each and 70 students have ordered them. A survey tells him that by decreasing the price by \$2, 5 more students would purchase the sweater. If Mr. Johnston wants to maximize income,
- what price should he sell the sweater?
 - how much total income will he earn?

Income = Price \cdot # of Students

x = # of times the price decreased

$$= (52 - 2x) \cdot (70 + 5x)$$

$$= 3640 - 140x + 260x - 10x^2 = 3640 + 120x - 10x^2$$

$$h = \frac{-b}{2a} = \frac{-120}{2(-10)} = \underline{6} \leftarrow x \text{ value at max income}$$

a) Price = $52 - 2(6) = \boxed{\$40}$

b) Income = $(40) \cdot (70 + 5(6)) = 40 \cdot 100 = \boxed{\$4000}$

4. Lord Byng is constructing a garden next to the school. The garden will have two identical rectangular sections, divided and surrounded by a fence, as shown below. No fencing is needed against the school. If Lord Byng has 300m of fencing to use:

- what is the **total** maximum area that can be enclosed?
- what are the dimensions of the **total enclosed area**?



$$A = x \cdot 2y$$

$$3x + 2y = 300$$

$$A = x \cdot 2 \left(\frac{300 - 3x}{2} \right)$$

$$2y = 300 - 3x$$

$$y = \frac{300 - 3x}{2}$$

$$A = x(300 - 3x)$$

$$= 300x - 3x^2$$

$$y = \frac{300 - 3(50)}{2}$$

$$= \frac{150}{2} = \underline{75m}$$

$$h = \frac{-b}{2a} = \frac{-300}{2(-3)} = \underline{50m} \leftarrow x \text{ value at max area}$$

c) $A = (50) \cdot 2(75) = 50 \cdot 150 = \boxed{7500m^2}$

b) $\boxed{50m \times 150m}$