

Warm-up

A LBSS musical theatre show has 4 unique singing roles and 3 unique dancing roles. If 7 singers and 9 dancers audition, how many cast lists are possible using:

i) FCP

$$\underbrace{7 \times 6}_{\text{sing}} \times \underbrace{5 \times 4}_{\text{Dance}} \times \underbrace{9 \times 8 \times 7}_{\text{Dance}}$$
$$= 423,360$$

ii) nPr

$$\begin{matrix} \text{sing} \\ 7P4 \end{matrix} \times \begin{matrix} \text{Dance} \\ 9P3 \end{matrix}$$
$$= 423,360$$

Ex 1 3 grade 8's and 4 grade 12's
want to line up for a photo at
grade 8 camp. How many ways
can they line up if:

a) No restrictions?

$$\begin{aligned} & \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \\ & = 7! = {}_7P_7 \quad \leftarrow 7 \text{ things, arranging } \underline{\text{all}} \text{ of them} \\ & = \boxed{5040} \end{aligned}$$

b) No students of same grade
beside each other? (FCP)

$$\begin{aligned} & \underline{\text{Gr}}_{12} \quad \underline{\text{Gr}}_8 \quad \underline{\text{Gr}}_{12} \quad \underline{\text{Gr}}_8 \quad \underline{\text{Gr}}_{12} \quad \underline{\text{Gr}}_8 \quad \underline{\text{Gr}}_{12} \\ & \underline{(4)} \times \underline{(3)} \times \underline{(3)} \times \underline{(2)} \times \underline{(2)} \times \underline{(1)} \times \underline{(1)} \\ & = 144 \end{aligned}$$

(nPr)

$$\begin{array}{ccccccccc} & \underline{Gr_{12}} & & \underline{Gr_{12}} & & \underline{Gr_{12}} & & \underline{Gr_{12}} & & \underline{Gr_{12}} \\ & & \underline{Gr_3} & & \underline{Gr_3} & & \underline{Gr_3} & & \underline{Gr_3} & \\ \Rightarrow & \underline{(4)} & \times & \underline{(3)} & \times & \underline{(2)} & \times & \underline{(1)} & = & 4! \text{ or } 4P_4 \\ & & \underline{(3)} & \times & \underline{(2)} & \times & \underline{(1)} & = & 3! \text{ or } 3P_3 \end{array}$$

$$\Rightarrow 4P_4 \times 3P_3 = 144$$

Ex 2 How many ways can we rearrange the letters of 'PENCILS' if:

a) No restrictions?

$$\begin{array}{cccccccc} \underline{7} & \underline{6} & \underline{5} & \underline{4} & \underline{3} & \underline{2} & \underline{1} & = 7! = 7P_7 \\ & & & & & & & = 5040 \end{array}$$

b) the letters 'PEN' must stay together in that order?

C I L S PEN | 'Letter' scrabble blocks

C I PEN L S

$$\underline{\textcircled{5}} \quad \underline{\textcircled{4}} \quad \underline{\textcircled{3}} \quad \underline{\textcircled{2}} \quad \underline{\textcircled{1}} = 5! = 5P_5$$
$$= \underline{\textcircled{120}}$$

c) the letters 'PEN' are together but not necessarily in that order?

~~PEN~~ I L S = 5!

5 4 3 2 1 [PEN]

P E N = 3!

$$5! \times 3! = 720$$

Order of big blocks order of 'PEN'

Ex 3 Alice, Bob, Colin, Denise, Erika, and Frank have tickets to a White Caps game, and they are sitting together (A1 - A6).

How many ways can they sit if:

a) No restrictions?

$$6! = 6P_6 = 720$$

b) Alice sits next to Frank

b) Alice and Denise are dating and they want to sit together?

$$\boxed{DA} \quad \boxed{BC} \quad \boxed{EF} \\ \underline{\textcircled{5}} \quad \underline{\textcircled{4}} \quad \underline{\textcircled{3}} \quad \underline{\textcircled{2}} \quad \underline{\textcircled{1}} = 5! = 5^P_5$$

$$\boxed{AD} \text{ rearrange} = 120$$

$$2! \Rightarrow 5! \cdot 2! = \boxed{1240}$$

c) just before game day Alice and Denise break up, and they don't want to sit together?

$$\begin{array}{l} AD \\ \text{sit} \\ \text{together} \end{array} \textcircled{\text{or}} \begin{array}{l} AD \\ \text{Don't sit} \\ \text{together} \end{array} = \begin{array}{l} \text{total} \\ \# \text{ of} \\ \text{possibilities} \end{array}$$

$$\cancel{240} + \overset{-240}{X} = 720 \overset{-240}$$

$$X = 720 - 240$$

$$X = \boxed{480}$$