c) at least two flags are used?

1 odd
no zero

0 allowed
2 digits taken away

14. a) How many odd six digit numbers have no repeating digits?

8 8 7 6 5 5 = 67,200

odd

b) Consider the question "How many even six digit numerals have no repeating digits?" Explain why we need to consider two separate cases to determine the answer.

1 zero at end 2 no zero at end

c) How many even six digit numerals have no repeating digits?

15. In the final of a 100-metre race there are 8 competitors. The number of possible ways in which the gold, silver, and bronze medals can be awarded is

A. 21 B. 24

C. 336 D. 512

...ow many even 5-digit whole numbers are there? Note that 31248 is acceptable, but ...1248 is not.

... 13 776

... 15 120

... 45 000

... 50 000

...andra is taking an examination which consists of two parts, A and B, with the following ...structions.

- Part A consists of three questions and the student must do two.

1 zero at end

no zero

no zero
no repeat

0

9 8 7 6 5 1

= 15,120

2 Not zero at end

no zero
no repeat

no repeats
×2

no zero
even (2,4,6,8)

8 8 7 6 5 4

= 53,760

15,120 + 53,760

= 68,880

Warm-up

Sarah is part of a family of 8 (including herself). If they line up next to eachother for a family photo, how many ways can they line up if:

a) all 8 members are in the photo?

$$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40,320$$

b) they take photos of 5 members at a time?

$$\underset{\#1}{8}\ \ \underset{\#2}{7}\ \ \underset{\#3}{6}\ \ \underset{\#4}{5}\ \ \underset{\#5}{4} = 6720$$

c) they take photos of at most 4 members at a time?

$8 \times 7 \times 6 \times 5$ 4 ppl

or

$+\ 8 \times 7 \times 6$ 3 ppl

or

$+\ 8 \times 7$ 2 ppl

or

$+\ 8$ 1 ppl

$= 2080$

Ex Later that year, Sarah has a family reunion with 20 total people (including herself). If all 20 line up for the photo, how many arrangements are possible?

$\underline{20}\ \underline{19}\ \underline{18}\ \underline{17}\ \underline{16}\ \ldots\ \underline{3}\ \underline{2}\ \underline{1}$

$= 20!$ Factorial Symbol

$= 2.432902008 \times 10^{18}$ move the decimal back 18 times

$\approx 2432902008000000000$

Seconds since start of universe

14.7 billion years $\approx 4.32 \times 10^{17}$ s

Factorials happen a lot in these problems

Warmup a) $\underline{8}\times\underline{7}\times\underline{6}\times\underline{5}\times\underline{4}\times\underline{3}\times\underline{2}\times\underline{1} = 8!$

We say "8 factorial"

b) $\underline{8}\times\underline{7}\times\underline{6}\times\underline{5}\times\underline{4} = \frac{8!}{3!} = \frac{8\times7\times6\times5\times4\times\cancel{3}\times\cancel{2}\times\cancel{1}}{\cancel{3}\times\cancel{2}\times\cancel{1}}$

You can break up factorials

$5! = 5\times4\times\overbrace{3\times\underbrace{2\times1}_{2!}}^{3!}$

$5! = 5\times4\times3\times2!$

$5! = 5\times4\times3!$

$5! = 5\times4!$

This can help simplify fractions with factorials (which shows up a lot)

Ex

$$\frac{12!}{10!} = 132$$

$$\frac{12 \times 11 \times \cancel{10!}}{\cancel{10!}} = 12 \times 11 = 132$$

Ex

$$\frac{8!}{5!\,3!} = \frac{8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!}\,3!}$$

$$\frac{8 \times 7 \times 6}{3!} = \frac{8 \times 7 \times \cancel{6}}{\cancel{3 \times 2} \times 1} = 56$$

In general,

$$n! = n \times (n-1) \times (n-2) \times (n-3) \times \dots \times 3 \times 2 \times 1$$

$$n = 1, 2, 3, 4, \dots$$

(It turns out we need $0! = 1$,
so we define $0! = 1$)

Permutations vs. Combinations

So far, we have always been talking about permutations.

Permutation is an arrangement where the order matters.

Ex family photos, Bank PIN
License plates, words

Combination is an arrangement where the order doesn't matter

Ex Subway toppings, hand of cards
groups of people

For permutations (order matters),
if you have 'n' unique things and you want to rearrange them all, then the total number of arrangements

$= n!$ (See warm-up a))

If you have 'n' unique thing, but you only want to rearrange 'r' of them at once, the total number of arrangements $= \dfrac{n!}{(n-r)!}$ (See warm-up b))

Ex Using the letters of 'KELOWNA', how many permutations are possible if:

a) we make 7 letter 'words'

$$\Rightarrow n! = 7!$$

$$= \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$$

b) we make 4 letter words

$$\Rightarrow \frac{n!}{(n-r)!}$$

$n = 7$ (7 letters total)

$r = 4$ (4 letters to rearrange)

$$\Rightarrow \frac{7!}{(7-4)!} = \frac{7!}{3!} = \underline{840}$$

$$= \underline{7} \times \underline{6} \times \underline{5} \times \underline{4}$$

$${}_nP_r = \frac{n!}{(n-r)!}$$

in Calculator ${}_7P_4$

$\boxed{7}\ \boxed{nPr}\ \boxed{4} = 840$

Ex A Tech Company has employee ID codes. The ID code is 2 different odd numbers, followed by 4 different consonants (Y is a vowel). How many ID codes are possible?

a) FCP

NO A E I O U Y

odd	odd	let	let	let	let
5	4	20	19	18	17

= 2,325,600

b) Factorials

2 odd and 4 consonants

$${}_{5}P_{2} \times {}_{20}P_{4} = 2{,}325{,}600$$