1. How many different arrangements can be made using all the letters of each word?

a. CHAPTER

7 unique letters

$7! = 5040$

b. TABLETOP

8 letters
2 T's

$\frac{8!}{2!} = 20160$

c. PARALLEL

8 letters
2 A's 3 L's

$\frac{8!}{3!2!} = 3360$

2. Mr. Gustainis has made a multiple-choice test with 11 questions. Each question can be answered A, B, C, or D. How many different ways could you fill out the test (no blanks) if:

A, B, C, D

a. no restrictions?

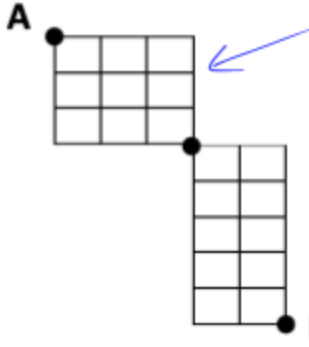
$\underset{\#1}{4} \times \underset{\#2}{4} \times \underset{\#3}{4} \times \ldots \times \underset{\#9}{4} \times \underset{\#10}{4} \times \underset{\#11}{4} = 4^{11} = 4\,194\,304$

b. Mr. G tells you there are 3 A's, 4 B's, 1 C's and the rest D's in the correct answer?

AAA BBBB C DDD - 11 letters, 3A's, 4B's, 3D's

$\frac{11!}{3!4!3!} = 46{,}200$

3. How many pathways are possible from A to B if the paths must always move closer to B?



3R 3D $\Rightarrow \frac{6!}{3!3!} = 20$

2R 5D $\Rightarrow \frac{7!}{2!5!} = 21$

AND

total $= 20 \times 21 = 420$

4. How many 6-card hands contain:

a. 3 red cards and 3 spades?

$26C_3 \times 13C_3$

$= 743600$

b. exactly 2 Jacks but no aces?

$4C_2 \times 44C_4$ (52 − 4J − 4A = 44 left)

$= 814506$

c. at most 2 Kings?

52 − 4 no K

OK 0 of K ⊕ 1 K 5 of O ⊕ 2 K 4 of O

$= 48C_6 \oplus 4C_1 \cdot 48C_5 \oplus 4C_2 \cdot 48C_4$

$= 12271512 \oplus 6849216 \oplus 1167480$

$= 20,288,208$

d. at least 4 Diamonds?

4D 2 of O ⊕ 5D 1 of O ⊕ 6D

Diamonds: 52 − 13 = 39

$= 13C_4 \cdot 39C_2 \oplus 13C_5 \cdot 39C_1 \oplus 13C_6$

$= 529815 \oplus 50193 \oplus 1716$

$= 581,724$

5. Mr. G is putting together a Physics Olympics team of 10 students. There are 13 grade 11s and 8 grade 12s that want to compete. How many different ways could he select his team if:

10 / 1

a. there is an equal number of grade 11s and grade 12s on the team?

$13C_5 \times 10C_5 = 324,324$

b. Eugene (grade 11) and exactly 2 other grade 11s are on the team?

$1C_1 \times 12C_2 \times 10C_7 = 7920$

Eugene — Gradell's (No Eugene) — Grade 12's

c. At least 2 grade 11s are on the team (*Solve using the complement for full marks*)?

total = 0 Gr11; 10 Gr12 ⊕ 1 Gr11; 9 Gr12 ⊕ 2 Gr11; 8 Gr12 ⊕ ... ⊕ 9 Gr11; 1 Gr12 ⊕ 10 Gr11; 0 Gr12

X

X = total − 0 Gr11; 10 Gr12 − 1 Gr11; 9 Gr12 (at least 2 Gr 11's)

$= 23C_{10} - 10C_{10} - 13C_1 \cdot 10C_9 = 1,143,935$