Permutations with Repetitions

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${}_nP_r$ → requires that all the objects are <u>unique</u>.

<u>Eg.</u> A math, physics and chem

a) textbook are on a shelf. How many arrangements?

$\underline{3}\ \ \underline{2}\ \ \underline{1} = {}_3P_3 = 3! = \underline{6}$

MPC, PMC, CMP, PCM, MCP, CPM

b) If we replace the chem text book with another identical physics book, now how many arrangements?

MP~~C~~ P, PM~~C~~ P, ~~C~~ P MP, P~~C~~ P M, M~~C~~ P P, ~~C~~ P PM

MPP, PMP, PMP, PPM, MPP, PPM

only 3 arrangements are left

$$= \frac{6}{2!}$$

6 ← total ${}_nP_r$

2! ← # of duplicates

Part 2

In lesson 2, we investigated the number of arrangements of all of the letters in a particular word. In every case, the letters in the word were different. In this part, we investigate what happens when there are letters which repeat within the same word.

To examine this scenario, consider the following four letter permutations of a word without repetitive letters, ROSE. Notice there are ${}_4P_4$, or 24 different arrangements.

ROSE	REOS	OSRE	SROE	SERO	EORS
ROES	RESO	OSER	SREO	SEOR	EOSR
RSOE	ORSE	OERS	SORE	EROS	ESRO
RSEO	ORES	OESR	SOER	ERSO	ESOR

a) Now, if we change the E in ROSE to an S, we get $\boxed{\text{ROSS}}$, a word with two letters which are repeating. If we change all the E's in the above list to S's, we will get all the arrangements for ROSS as shown in the list below.

ROSS	~~RSOS~~	OSRS	SROS	SSRO	~~SORS~~
~~ROSS~~	~~RSSO~~	OSSR	SRSO	SSOR	~~SOSR~~
RSOS	ORSS	~~OSRS~~	SORS	~~SROS~~	~~SSRO~~
RSSO	~~ORSS~~	~~OSSR~~	SOSR	~~SRSO~~	~~SSOR~~

i) Complete the last column.

ii) There are no longer 24 different arrangements. Arrangements like ROSE and ROES from the first list both become ROSS in the second list and count as only **one arrangement**. The number of different arrangements of ROSS is 12.

iii) Notice that there are $\frac{1}{2}$ or $\frac{1}{2!}$ as many permutations of ROSS as there are of ROSE.

Hence, the number of permutations of ROSS is $\frac{4!}{2!}$, or $\frac{24}{2} = 12$.

b) If we change the O and E in ROSE to S, we get RSSS, a "word" with three repeating letters, with the arrangements shown below.

RSSS	~~RSSS~~	SSRS	~~SRSS~~	~~SSRS~~	~~SSRS~~
~~RSSS~~	~~RSSS~~	SSSR	~~SRSS~~	~~SSSR~~	~~SSSR~~
~~RSSS~~	SRSS	~~SSRS~~	~~SSRS~~	~~SRSS~~	~~SSRS~~
~~RSSS~~	~~SRSS~~	~~SSSR~~	~~SSSR~~	~~SRSS~~	~~SSSR~~

i) Complete the last column.

ii) Arrangements like ROSE, ROES, RSOE, RSEO, REOS, and RESO from the first list all become one arrangement of RSSS. The 24 original arrangements of ROSE is now reduced to 4 arrangements of RSSS.

iii) Notice that there are $\frac{1}{6}$ or $\frac{1}{3!}$ as many permutations of RSSS as there are of ROSE.

Hence, the number of permutations of RSSS is $\frac{4!}{3!}$, or $\frac{24}{6} = 4$

If we want to arrange 'n' objects where 'a' are the same of one type, 'b' are the same of another type, and 'c' that are the same of another type...

$$\text{total number} = \frac{n!}{a!\,b!\,c!}$$

total with no repeats

number of repeats

Ex How many we rearrange the letters of:

a) CANADA

3A's , 6 total

$$\Rightarrow \frac{6!}{3!} = \boxed{120}$$

b) BASEBALL

2 B's, 2 A's, 2L's

8 total

$$\frac{8!}{2!\,2!\,2!} = \boxed{5040}$$

$\boxed{8}\,\boxed{!}\,\boxed{\div}\,\boxed{(}\,\boxed{2}\,\boxed{!}\cdot\boxed{2}\,\boxed{!}\cdot\boxed{2}\,\boxed{!}\,\boxed{)}$

<u>Ex</u> Mr. G has a box of donuts with 3 Boston Cream, 2 chocolate, 5 maple dip, 1 plain and 1 Blueberry. If Mr. G eats 1 each day, how many different orders could he eat them in?

___ ___ ___ . . . . ___ (12 donuts)

day 1 day 2 day 3 ... day 12

FCP <u>does not</u> work because we have duplicates.

$$\frac{12!}{3!\,2!\,5!} = \boxed{332{,}640}$$

$3!$ ↑ Boston, $2!$ ↑ Choc, $5!$ ↑ Maple

$\boxed{\#1\text{-}5}$