

Warmup

How many 5-card hands have:

a) exactly 2 Diamonds?

$$\begin{array}{c} 13 \text{ } ^C_2 \\ \uparrow \\ \text{Diamonds} \end{array} \times \begin{array}{c} 39 \text{ } ^C_3 \\ \uparrow \\ \text{NO Diamonds} \end{array}$$

$$= \underline{712,842}$$

b) exactly 1 King
and no Sevens?

$$\begin{array}{c} 4 \text{ } ^C_1 \\ \uparrow \\ \text{Kings} \end{array} \times \begin{array}{c} 44 \text{ } ^C_4 \\ \uparrow \\ \text{NO Sevens } (-4) \\ \text{NO King } (-4) \end{array}$$

$$= \underline{543,004}$$

Schedule:Mon: Review of #4-6wed: Quiz #4-6 (include #1-3)After: #7 - Combining everything
Project work time

#6 / 'At least' and 'at most'

Ex 1 How many 5-Card hands contain:

a) at most 1 king?

1K, 4 other (or) 0K, 5 other

$$= 4 \binom{48}{1} \times 48 \binom{48}{4} + 48 \binom{48}{5}$$

No kings No kings

(52-4) = (51-3)

$$= 778,320 + 1,712,304$$

$$= \underline{2,490,624}$$

b) at least 1 club?

1c, 4 other (or) 2c, 3 other (or) 3 club, 2 other
(or) 4 club, 1 other (or) 5c = X

Use Complement

total possibilities = 0c, 5 other (or) 1c, 4 other (or) 2c, 3 other
(or) 3c, 2 other (or) 4c, 1 other (or) 5c

$$\text{total} = 0c, 5\text{ other} + X$$

$$\text{total} - 0c, 5\text{ other} = X$$

$$52^5 - 39^5 = X$$

NO
↑
restrictions

NO clubs
(52-13)

$$2,598,960 - 575,757 = x$$

$$\text{at least 1 club} = \underline{2,023,203}$$

Ex 2

A Car collector has 10 cars.

- a) For an exhibition she chooses 8 to display. How many different ways can she choose the cars?

$$10^C_8 = 45$$

- b) For a family trip she chooses 2 to take. How many ways can she

to take. How many ways can she choose the cars?

$$10C_2 = 45$$

a) Picked
8 cars

Left over
2 cars

b) Picked
2 cars

Left over
8 cars

$$10C_2 = \frac{10!}{(10-2)! \cdot 2!} = \frac{10!}{8! \cdot 2!}$$

$$10C_8 = \frac{10!}{(10-8)! \cdot 8!} = \frac{10!}{2! \cdot 8!}$$

Same

Ex 3

After a business meeting, everyone shakes hands with each other person. If 300 hand shakes happened, how many people were at the meeting?

20 people: $\textcircled{20} C_2 = 190$

↑ each pair shakes hands ↑ hand shakes

Guess and check • Bigger than 20 $23 C_2 = 253$

Bigger than 23 $25 C_2 = 300$

25 people