

Dice \rightarrow Foundations 12 – Introduction to Probability Part 1

Homework: Lesson #1 on Pg. 123: #1-11, Skip #5c

Ex 1

A six-sided die is rolled.

- a) List all the possible outcomes (called the **Sample Space**) \leftarrow all

$\{1, 2, 3, 4, 5, 6\}$

- b) List all the outcomes that result in a number bigger than 2 (called an **Event**) \leftarrow might not be all outcomes

$\{3, 4, 5, 6\}$

(The book calls events a **subset** of the sample space)

- c) Assuming it's a fair die (all outcomes equally likely), what is the **probability** that you roll a number bigger than 2?

Bigger than 2	Not Bigger than 2
3	1, 2
4	
5	
6	

$$\frac{4 \text{ outcomes bigger than 2}}{6 \text{ total outcomes}} = \frac{4}{6} = \frac{2}{3} = 0.666\dots$$

$$= 66.666\dots\%$$

$$\frac{\# \text{ of favorable outcomes}}{\text{total } \# \text{ of outcomes}} = 66.7\%$$

If all events are equally likely: $P(\text{some event}) =$

For any event: $0\% \leq P(\text{any event}) \leq 100\%$ \rightarrow same

Or $0 \leq P(\text{any event}) \leq 1$

- d) List the **event** "a number that is not bigger than 2" then find the probability of rolling a number that is not bigger than 2

$\{1, 2\}$

$$\frac{2^{\text{outcomes not bigger than 2}}}{6 \text{ total}} = \frac{2}{6} = \frac{1}{3} = 0.333\dots = 33.3\%$$

The events in b) and d) are called **complementary** events.

We can write $P(\text{not bigger than 2}) = P(\text{bigger than 2})$ \leftarrow opposite, complement

$P(\text{bigger than 2}) + P(\text{not bigger than 2}) = 100\% \text{ or } 1$

- e) List the event "a number bigger than 8" then find the probability of rolling a number bigger than 8

$$\{\emptyset\} = \{\emptyset\} \quad \text{nothing} \quad \frac{0}{6} = 0\%$$

If $P(\text{some event}) = 0\%$ we call the event impossible

- f) List the event "a number less than 7" then find the probability of rolling a number less than 7

$$\{6, 5, 4, 3, 2, 1\} \Rightarrow \frac{6}{6} = 100\%$$

If $P(\text{some event}) = 100\%$ we call the event Certain
Guaranteed

Ex 2

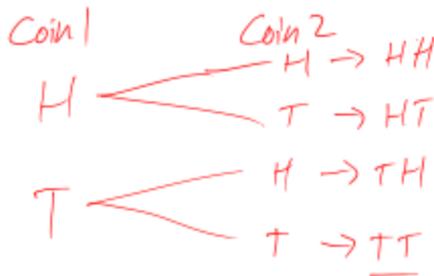
Jimmy flips two coins at the same time and wants to determine the probability of flipping both heads. He lists the sample space as (2 heads, 2 tails, 1 tail and 1 head). Since only 1 of the three outcomes is favorable, he determines the probability to be $\frac{1}{3}$.

- a) Explain why Jimmy's reasoning is not correct

Coin 1	Coin 2	
H	H	→ HH - 1
T	T	→ TT - 1
H	T	→ HT - 2
T	H	→ HT - 2

Not all outcomes are equally likely

- b) Use a tree diagram to find the probability for each outcome (2 heads, 2 tails or 1 head 1 tail)



$$P(2 \text{ heads}) = \frac{1}{4} = 0.25 = 25\%$$

$$P(2 \text{ tails}) = \frac{1}{4} = 0.25 = 25\%$$

$$P(1T, 1H) = \frac{2}{4} = 0.5 = 50\%$$

Ex 3

Instead of using tree diagrams, a table can be really helpful for listing out the **sample space** (all possible outcomes). An incomplete sample is shown on the side for two 6-sided dice.

- a) Fill out the table to show the sample space
 b) How many outcomes are in the sample space?

$$\frac{6}{\text{Die 1}} \cdot \frac{6}{\text{Die 2}} = 36$$

- c) List the event:
 "the same number appears on both dice"

$$\{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$$

- d) Find the probability that the same number appears on both dice

$$\frac{6}{36} = \frac{1}{6} = 16.7\%$$

opposite (with arrow pointing to the complement calculation)

- e) Find the probability that different numbers appear on each die using the complement

$$\frac{30}{36} = \frac{5}{6} = 83.3\%$$

$$100\% - 16.7\% = 83.3\%$$

	Die #1					
Die #2	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

With your partner, pick 2 differently shaped dice from the bag and do a) to e).

D8, D20

$$b) 8 \cdot 20 = 160$$

$$c) \{(1,1), (2,2), \dots, (8,8)\}$$

$$d) \frac{8}{160} = \frac{1}{20} = 5\%$$

$$e) 100\% - 5\% = 95\%$$

D10, D6

$$b) 6 \cdot 10 = 60$$

$$c) \{(1,1), (2,2), \dots, (6,6)\}$$

$$d) \frac{6}{60} = \frac{1}{10} = 10\%$$

$$e) 100\% - 10\% = 90\%$$