2.A \$20 bill, two \$10 bills, three \$5 bills and four \$1 bills are placed in a bag. If a bill is chosen at random, what is the expected value for the amount chosen?
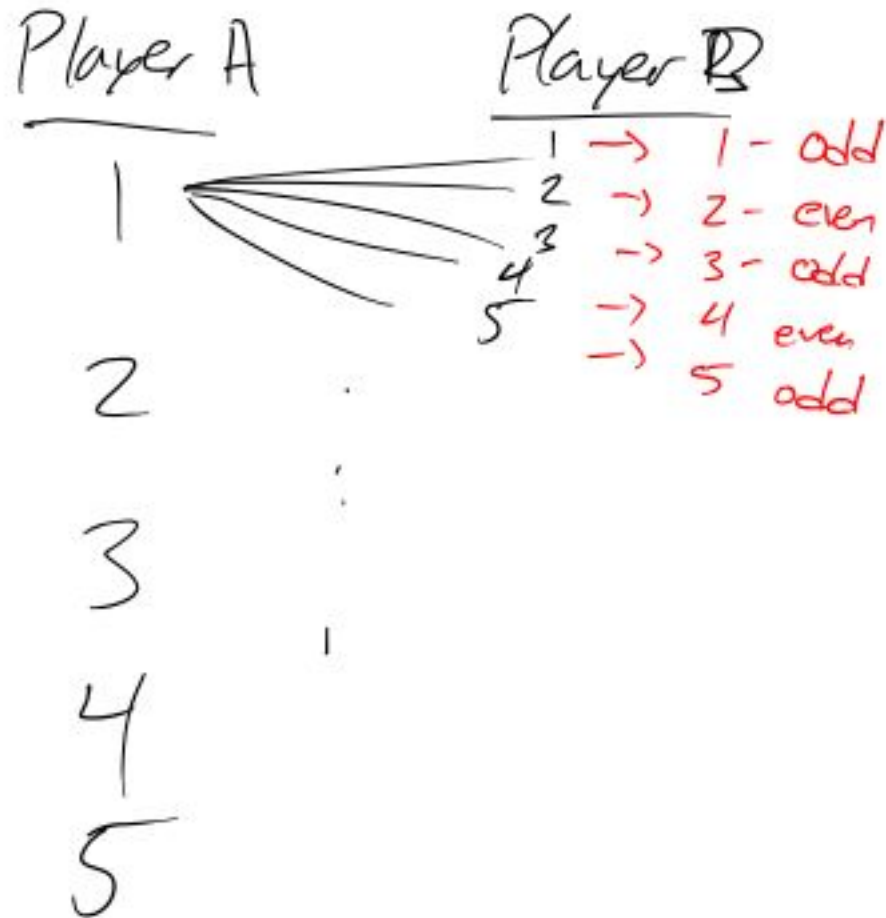
$$\$20 \qquad \$10 \qquad \$5 \qquad \$1$$

$$EV = \$20 \cdot \frac{1}{10} + \$10 \cdot \frac{2}{10} + \$5 \cdot \frac{3}{10} + \$1 \cdot \frac{4}{10}$$

1 \$20

2 \$10 $\rightarrow$ 10 bills

3 \$5   total

4 \$1

$$= \$5.90$$

3. As we discussed last class, fair games have payouts that match the odds against winning. For example, if you bet on rolling a 3 on a die, the odds against rolling a 3 are 5 : 1. So if you bet \$1, you should get your money

4. In a game, you flip a coin twice and record the number of heads that occur. You get 10 points for 2 heads, zero points for 1 head, and 5 points for no heads. What is the expected value for the number of points you'll win per turn?

Flip 1   Flip 2

$$H < \begin{array}{c} H - HH \\ T - HT \end{array}$$

$$T < \begin{array}{c} H - TH \\ T - TT \end{array}$$

4 outcomes

$$2H \quad 1H \quad 0H$$

$$EV = 10 \cdot \frac{1}{4} + \bigotimes_{0 pes} + 5 \cdot \frac{1}{4}$$

$$= 3.75 \, pts$$

5. There is an equally likely chance that a falling dart will land in any square on the rug below. The following system is used to find the number of points the player wins. What is the expected value for the number of points won?

8. Integer Game: 2 players choose an integer from 1 to 5. If the (product) MULT of the two integers is even, then Player A scores 5 points and Player B loses 2 points. If the product of the two integers is odd, then Player B scores 5 points and Player A loses 2 points. Find the expected value of each player.

Player A

1

2

3

4

5

Player B

1 → 1 - odd
2 → 2 - even
3 → 3 - odd
4 → 4 even
5 → 5 odd

$$Ev = 5 \cdot \overset{even}{\boxed{}} i + -2 \cdot \overset{odd}{\boxed{}}$$

$$PA =$$

$$Ev = -2 \cdot \overset{even}{\boxed{}} i + 5 \cdot \overset{odd}{\boxed{}}$$

$$PB =$$

9. An airline is considering adding a route to the city of New Orleans, Louisiana. Market research predicts that if the airline serves New Orleans, there is a 42% probability of making a $700,000 profit, a 32% probability of

9. An airline is considering adding a route to the city of New Orleans, Louisiana. Market research predicts that if the airline serves New Orleans, there is a 42% probability of making a $700,000 profit, a 22% probability of breaking even, and a 36% probability of losing $1,000,000. What is the expected value of adding a route to New Orleans?

Profit   even   loss

$$EV = 0.42 \cdot \$700,000 + 0.22 \cdot \$0 + 0.36 \cdot -\$1,000,000$$
$$= \boxed{-\$66,000}$$

10. A landscaper mows 25 lawns per day on sunny days and 15 lawns per day on cloudy days. If the weather is sunny 65% and cloudy 35% of the time, how many lawns can he expect to mow per day?

Sunny   Cloudy

$$EV = 25 \cdot 0.65 + 15 \cdot 0.35 = 21.5 \text{ lawns per day}$$

X times

11. You are playing a number cube game where you need 60 points to win. On each turn you roll a pair of dice (6-sided number cube). If you roll doubles, your score is the product of the numbers. If you do not roll doubles, you do not score any points. Find the expected value of each turn. How many turns will it take on average to score 60 points?

#7

(1,1) (2,2) (3,3) (4,4) (5,5) (6,6)

$$EV = (1) \cdot \frac{1}{36} + 4 \cdot \frac{1}{36} + 9 \cdot \frac{1}{36} + 16 \cdot \frac{1}{36} + 25 \cdot \frac{1}{36} + 36 \cdot \frac{1}{36}$$
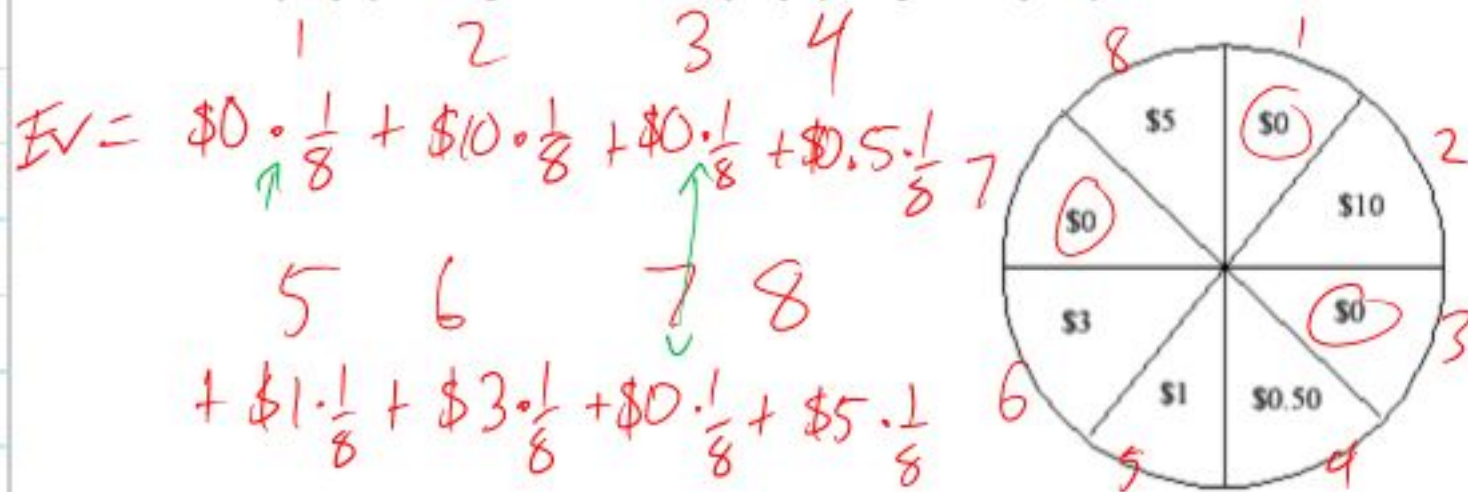
$$= 2.5278 \text{ pts per game (av)}$$

$$\frac{60}{2.5278} = 23.7 \text{ turns} \rightarrow 24 \text{ turns}$$

Same

12. You can pay $2 at a local carnival and spin the spinner at right. The spinner is divided into 8 congruent sections. The section on which the spinner lands shows the dollar amount you win. What is the expected value

6

Same

12. You can pay $2 at a local carnival and spin the spinner at right. The spinner is divided into 8 congruent sections. The section on which the spinner lands shows the dollar amount you win. What is the expected value for each time you play? Is the game fair? Would you play this game? Explain your decision.

$$EV = \$0 \cdot \frac{1}{8} + \$10 \cdot \frac{1}{8} + \$0 \cdot \frac{1}{8} + \$0.5 \cdot \frac{1}{8}$$

$$+ \$1 \cdot \frac{1}{8} + \$3 \cdot \frac{1}{8} + \$0 \cdot \frac{1}{8} + \$5 \cdot \frac{1}{8}$$

$$= \$2.4375 > \$2$$

Good game to play

Ex 1 The table below shows how students at Byng generally get to school.

a) Complete the totals in the chart

b) How many students attend Lord Byng?

1200

	Bus, B	Car, C	Other, O	Total
Byng Arts A	250	100	75	425
Regular R	400	275	100	775
Total	650	375	175	1200

c) Use the numbers in the table to determine:

i. P(R)

$$= \frac{175}{1200}$$

$$= 64.58\%$$

ii. P(R ∩ C)

$$= \frac{275}{1200}$$

$$= 22.92\%$$

take a Car Given that they are R

iii. P(C|R)

$$= \frac{275}{775} \leftarrow \text{total R}$$

$$= 35.48\%$$

iv. P(R|C) ← Regular given that table Car

$$= \frac{275}{375} \leftarrow \text{total take Car}$$

$$= 73.33\%$$

d) If a student is chosen at random, determine the probability that they are:

i. in Byng Arts

$$= \frac{425}{1200}$$

$$= 35.42\%$$

ii. in Byng Arts and take the bus

$$= \frac{250}{1200}$$

$$= 20.8\%$$

iii. in Byng Arts or take the bus

$$= \frac{75 + 100 + 250 + 400}{1200}$$

$$= \frac{825}{1200} = 68.75\%$$

e) Are the events "the student is in Byng Arts" and "the student takes the bus" independent events? Explain

$$P(A) \overset{?}{=} P(A|B) \quad \leftarrow \overset{\text{just bus}}{\text{ppl}}$$

don't affect each other

prob Arts   ?

$$35.42\% \overset{?}{=} \frac{250}{650} = 38.46\%$$

Can also check

$$P(B) = P(B|A) \leftarrow \text{won't be equal because not independent}$$

not Same ⇒ not independent

2 days

2 days

Ex 2 During a weekend in Vancouver, there is a 65% chance it will rain and 35% chance it will be dry on each day. If the weather on each day is independent:

a) Draw a tree diagram to show all possible outcomes for the weather during the weekend

Sat   Sunday   AND

35% Sun
- 35% Sun ← 35% · 35% = 12.25%
- 65% rain ← 35% · 65% = 22.75%

65% Rain
- 35% Sun ← 65% · 35% = 22.75%
- 65% rain ← 65% · 65% = 42.25%

b) Use the tree diagram to determine the probability that:

total 100% ← good to check

i. It rains both days

$$42.25\%$$

ii. it rains one day

$$22.75\% + 22.75\%$$
$$= 45.5\%$$

iii. it rains at least 1 day

$$42.25 + 22.75 + 22.75$$
$$= 87.75\%$$

Ex 3 During a weekend in Vancouver, there is a 65% chance it will rain and 35% chance it will be dry on Saturday. If it rains on Saturday, the probability it rains on Sunday increases by 10%. If it is dry on Saturday, the probability it rains on Sunday is decreased by 15%.

a) Draw a tree diagram to show all possible outcomes for the weather during the weekend

Sat   Sunday

35% Sun
- 50% Sun ← 35% · 50% = 17.5%
- 65% - 15% = 50% rain ← 35% · 50% = 17.5%

65% rain
- 25% Sun ← 65% · 25% = 16.25%
- 65% + 10% = 75% rain ← 65% · 75% = 48.75%

total 100% ✓

b) Use the tree diagram to determine the probability that:

ii. It rains both days

$$48.75\%$$

ii. it rains one day

$$16.25 + 17.5$$
$$= 33.75$$

iii. it rains at least 1 day

$$100\% - 17.5\%$$
$$= 82.5\%$$