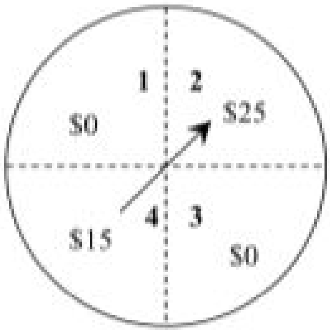
FOM 12 – Expected Value

The **expected value** of a game is defined as the sum of the products of each value of each outcome and the corresponding probability of the outcome. ← How much you will win (or lose) on average each time you play

Example:



The spinner above has four regions. The spinner is equally likely to land in any region. To find the expected value, follow these steps:

a. For each region, multiply the value of the prize (this example is a dollar amount) by the probability of landing on that prize. Put these values in the blanks.

b. Then add them up.

$\left(\$0 \cdot \frac{1}{4}\right)$ (region 1) + $\left(\$25 \cdot \frac{1}{4}\right)$ (region 2) + $\left(\$0 \cdot \frac{1}{4}\right)$ (region 3) + $\left(\$15 \cdot \frac{1}{4}\right)$ (region 4) = $10 (Expected Value) ← average winnings

To qualify for a **fair game**, the cost to play the game would be equal to the expected value. For this game to be fair, the cost to play would be $10 .

Ex 1 In a carnival game, players win prizes by rolling a cube. The cube has one red side, one white side, one blue side and three green sides. This game costs $1 to play. If the cube stops with the red face up, the player receives a prize worth 50 cents. If it stops with the white face up, the player wins a prize worth $1. When a blue face is showing, the prize is worth $1.50. If the cube shows a green face, the player wins nothing.

a. Verify that the game is not mathematically fair by calculating the expected value. Show all work below.

Red White Blue Green

$$EV = \$0.5 \cdot \frac{1}{6} + \$1 \cdot \frac{1}{6} + \$1.5 \cdot \frac{1}{6} + \cancel{\$0 \cdot \frac{3}{6}} = \$0.50$$

← less than $1

b. Adjust the cost of playing the game to make it fair.

Cost of play = $0.50

1. Imagine that you are the manager of a carnival. One of the game operators has designed a new game. In this game, players pick one card out of an ordinary deck of 52 playing cards. An ace wins $10, a face card (K, Q, or J) wins $1, and all other cards win nothing. Determine the cost to play this game in order to make it a fair game. Show work below.

Ace Face other

$$EV = \frac{4}{52}\cdot \$10 + \frac{12}{52}\cdot \$1 + \cancel{\frac{36}{52}\cdot \$0}$$

$$= \underline{\$1}$$

2.A $20 bill, two $10 bills, three $5 bills and four $1 bills are placed in a bag. If a bill is chosen at random, what is the expected value for the amount chosen?

3. As we discussed last class, fair games have payouts that match the odds against winning. For example, if you bet on rolling a 3 on a die, the odds against rolling a 3 are 5 : 1. So if you bet $1, you should get your money back and win an extra $5. If you roll any other number, you would lose your $1. Use expected value to show that this would be a fair game.

win lose

Bet $1 → money back, $5 extra or Lose $1

<u>Cost to play = $1</u>

Roll 3 Not 3

$$EV = \$6\cdot\frac{1}{6} + \cancel{\$0\cdot\frac{5}{6}}\ 0$$

$$= \underline{\$1}$$

<u>Cost to play = $0</u>

extra $5 Roll 3 Not 3 Lose $1

$$EV = \$5\cdot\frac{1}{6} + (-\$1)\cdot\frac{5}{6}$$

$$= \underline{\$0}$$