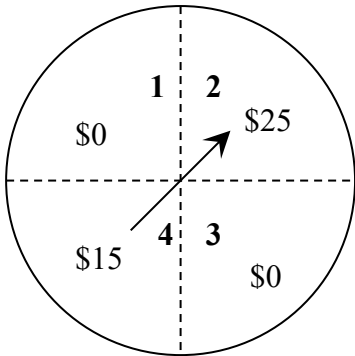


FOM 12 – Expected Value

The **expected value** of a game is defined as the sum of the products of each value of each outcome and the corresponding probability of the outcome.

Example:



The spinner above has four regions. The spinner is equally likely to land in any region. To find the expected value, follow these steps:

a. For each region, multiply the value of the prize (this example is a dollar amount) by the probability of landing on that prize. Put these values in the blanks.

b. Then add them up.

$$\frac{\quad}{\text{region 1}} + \frac{\quad}{\text{region 2}} + \frac{\quad}{\text{region 3}} + \frac{\quad}{\text{region 4}} = \frac{\quad}{\text{Expected Value}}$$

To qualify for a **fair game**, the cost to play the game would be equal to the expected value. For this game to be fair, the cost to play would be _____.

Ex 1 In a carnival game, players win prizes by rolling a cube. The cube has one red side, one white side, one blue side and three green sides. This game costs \$1 to play. If the cube stops with the red face up, the player receives a prize worth 50 cents. If it stops with the white face up, the player wins a prize worth \$1. When a blue face is showing, the prize is worth \$1.50. If the cube shows a green face, the player wins nothing.

a. Verify that the game is not mathematically fair by calculating the expected value. Show all work below.

b. Adjust the cost of playing the game to make it fair.

1. Imagine that you are the manager of a carnival. One of the game operators has designed a new game. In this game, players pick one card out of an ordinary deck of 52 playing cards. An ace wins \$10, a face card (K, Q, or J) wins \$1, and all other cards win nothing. Determine the cost to play this game in order to make it a fair game. Show work below.

2. A \$20 bill, two \$10 bills, three \$5 bills and four \$1 bills are placed in a bag. If a bill is chosen at random, what is the expected value for the amount chosen?

3. As we discussed last class, fair games have payouts that match the odds against winning. For example, if you bet on rolling a 3 on a die, the odds against rolling a 3 are 5 : 1. So if you bet \$1, you should get your money back and win an extra \$5. If you roll any other number, you would lose your \$1. Use expected value to show that this would be a fair game.

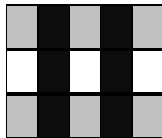
4. In a game, you flip a coin twice and record the number of heads that occur. You get 10 points for 2 heads, zero points for 1 head, and 5 points for no heads. What is the expected value for the number of points you'll win per turn?

5. There is an equally likely chance that a falling dart will land in any square on the rug below. The following system is used to find the number of points the player wins. What is the expected value for the number of points won?

Black = 40 points

Gray = 20 points

White = 0 points



6. A mysterious card-playing squirrel (pictured) offers you the opportunity to join in his game. The rules are:
To play you bet \$2.

If you pick a **spade** from a shuffled pack, you win get your money back and win an extra \$7.

If you don't, you lose the money you bet.

Find the expected value you win (or lose) per game.



7. A dice game involves rolling 2 dice. If you roll a 2, 3, 4, 10, 11, or a 12 you win \$6. If you roll a 5, 6, 7, 8, or 9 you lose \$6. Find the expected value you win (or lose) per game.

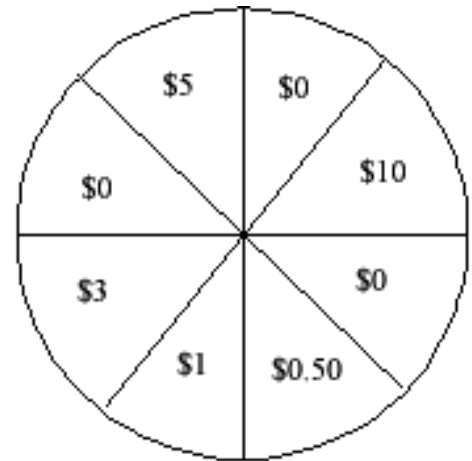
8. Integer Game: 2 players choose an integer from 1 to 5. If the product of the two integers is even, then Player A scores 5 points and Player B loses 2 points. If the product of the two integers is odd, then Player B scores 5 points and Player A loses 2 points. Find the expected value of each player.

9. An airline is considering adding a route to the city of New Orleans, Louisiana. Market research predicts that if the airline serves New Orleans, there is a 42% probability of making a \$700,000 profit, a 22% probability of breaking even, and a 36% probability of losing \$1,000,000. What is the expected value of adding a route to New Orleans?

10. A landscaper mows 25 lawns per day on sunny days and 15 lawns per day on cloudy days. If the weather is sunny 65% and cloudy 35% of the time, how many lawns can he expect to mow per day?

11. You are playing a number cube game where you need 60 points to win. On each turn you roll a pair of dice (6-sided number cube). If you roll doubles, your score is the product of the numbers. If you do not roll doubles, you do not score any points. Find the expected value of each turn. How many turns will it take on average to score 60 points?

12. You can pay \$2 at a local carnival and spin the spinner at right. The spinner is divided into 8 congruent sections. The section on which the spinner lands shows the dollar amount you win. What is the expected value for each time you play? Is the game fair? Would you play this game? Explain your decision.



Ex 1a. \$0.50 (less than \$1) **b.** \$0.50 **1.** \$1 **2.** \$5.90 **4.** 3.75 **5.** 24 **6.** +\$0.25 **7.** -\$2 **8.** A: 2.48 B: 0.52
9. -\$66,000 **10.** 21.5 **11.** 23.74 ~ 24 turns **12.** \$2.4375; Yes, because it is bigger than the price to play