Foundations 12 – Independent/Dependent Events

Homework: Lesson #4 on Pg. 152: #1-14

Warm-up

Blackjack is a game where you draw cards from a standard deck and add the values together to try and get as close as you can to 21 (without going over!). Each numbered card has a value equal to the number, face cards (J, Q, K) are worth 10 and Aces are worth 1 or 11 (whichever is better for you). So getting an Ace and a ten gets you 21 ("Blackjack")! Find:

a) P(First card is an Ace)

b) P(First card is a face card)

c) P(First card is a face card and second card is an ace)

d) Why can't you determine P(Second card is an ace) on its own?

If the probability of Event B depends on whether or not Event A has occurred, Events A and B are called ____________________ events.

If the probability of Event B DOES NOT depend on whether or not Event A has occurred, Events A and B are called ____________________ events.

Independent Events	**Dependent Events**

How could we change the rules of Blackjack so that every card drawn is an independent event?

Let's go back to Blackjack again. Let A = {first card is a face card} B = {second card is an ace}
When we calculated the probability for part c) of the warm-up, we used the fact that:

$$P(A \cap B) = P(A) \times P(B \text{ given that A happened})$$

We use the notation: P(B given that A happened) =

So the formula is: $P(A \cap B) =$

NOTE: $P(A \cap B) = P(B \cap A)$ *even though the calculations are different*

Ex 1

Two cards are drawn from a deck while playing Blackjack (**without** replacement). Use the formula to determine the probability of the following events:

a) Both cards are face cards

b) One card is a face card and the other is an ace (*different from the warmup!*)

Ex 2

Two cards are drawn from a deck **with** replacement. Determine the probability of the following events:

a) Both cards are face cards

b) One card is a face card and the other is an ace

When the two events, A and B, are independent:

$$P(B|A) =$$

So, with two independent events, the formula changes to:

$$P(A \cap B) =$$

It's really easy to confuse **mutually exclusive** and **independent/dependent** events.

- **Mutually exclusive** events involve whether two events can happen at the same time
- **Independent/dependent** events involve one event affecting the probability of another event

Are Event A and B…	**Mutually Exclusive?**	**Independent/Dependent?**
You pick a random Byng student A: They are over 6 feet tall B: They are over 200 pounds		
You draw a card from a deck A: It's a club B: It's a red card		

Useful formulae so far:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\qquad$ $P(A \cap B) = P(A) \bullet P(B|A)$

Ex 3

If $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, use appropriate formulas to determine:

a) $P(A \cap B)$

b) $P(B|A)$

c) $P(A|B)$

d) Are A and B mutually exclusive?

e) Are A and B independent?

Ex 4

Mr. G is doing a basketball drill where he shoots a free-throw then a 3-point shot. The probability that he hits the free-throw (Event A) is 0.6 and the probability he drains the 3-point shot (Event B) is 0.2. Assuming the shots are independent, determine:

a) $P(A \cap B)$

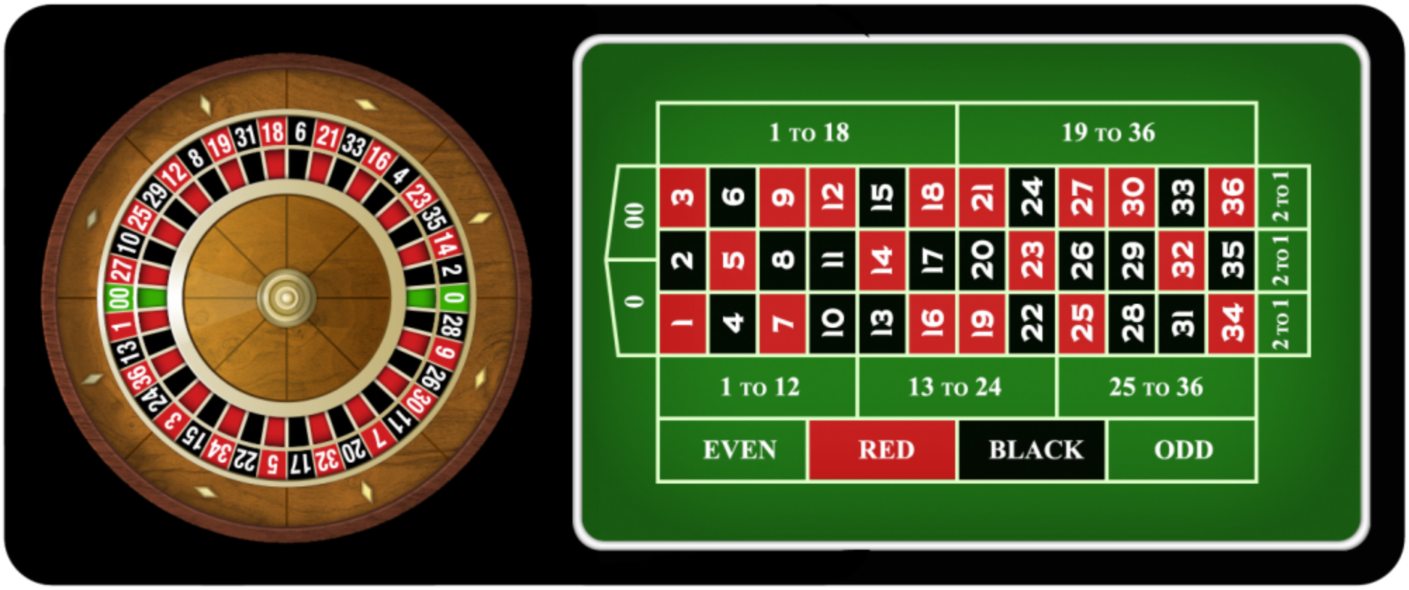
b) $P(A \cup B)$

c) $P(A' \cap B)$

d) $P(A' \cap B')$

Roulette

Another common casino game is Roulette. A ball is thrown onto a spinner and lands randomly on one of the numbers (0, 00, 1-36). The spinner and betting board is shown below.



You place bets on the board. Different bets have different payouts. You can bet on:

1. Single numbers
2. Red or Black (0 and 00 don't count as Red or Black)
   (For reference: 1 and 3 are reds, 2 and 4 are blacks)
3. Evens or Odds (0 and 00 not included)
4. Number range (1-12,13-24,25-36,1-18,19-36)
5. Columns (Left, middle or right)

Calculate the following:

a) P(Red) =

b) P(Red then another Red) =

c) P(10 Reds in a row) =

d) Mr. G thinks he's figured out how to win at Roulette reliably. He goes to the casino and waits until 9 Reds in a row happen at a table, then bets it all on Black, knowing that it is extremely unlikely for 10 Reds in a row to happen. What's wrong with his reasoning?

Mr. G and his friend Mr. H are playing Roulette. Let G = {Mr. G wins} and H = {Mr. H wins}

a) Mr. G bets on Red and Mr. H bets on Black. Are Mr. G winning and Mr. H winning:

   i. mutually exclusive? ii. independent?

b) Next round, Mr. G bets on Red again, while Mr. H bets on the middle column.

   i. P(G) ii. P(H)

   iii. $P(G \cap H)$ *(look at the picture)* iv. $P(G \cup H)$ *(use a formula)*

   v. $P(G|H)$ *(use a formula)* vi. $P(H|G)$ *(use a formula)*

   vii. Are G and H mutually exclusive? viii. Are G and H independent?

Fair payout odds are based on the odds against something happening. For instance, the odds against rolling a 2 on a 6 sided die is 5 : 1 (5 unfavorable, 1 favorable). So if you bet $1 on the die rolling a 2, you should win $5.

c) Find the fair payout odds for betting on Red in roulette

d) Find the fair payout odds for betting on the middle column

e) The casino payout odds for Red is 1 : 1 and for the middle column is 2 : 1. Why are the casino payout odds different from the fair payout odds?