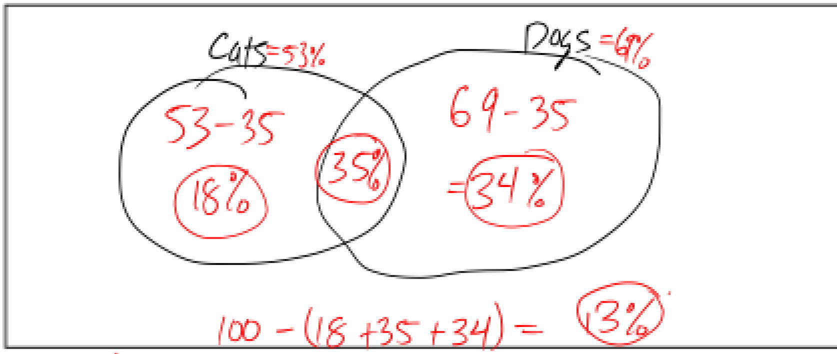
Warm-up

I polled the students in our class and got the following results. 53% of people said they liked cats, 69% said they liked dogs and 35% said they liked cats and dogs. Draw a Venn Diagram to figure out the probability that a person chosen at random:

i. likes just cats
ii. likes just dogs
iii. doesn't like cats or dogs
iv. likes cats or dogs (at least one)



i. 18%
ii. 34%
iii. 13%
iv. 100 - 13 = 87%

Ex 1

A six-sided die is rolled. Let's call Event A "An even number is rolled" and Event B "An odd number is rolled"

a) List all the possible outcomes for:

i. Event A

{2, 4, 6}

ii. Event B

{1, 3, 5}

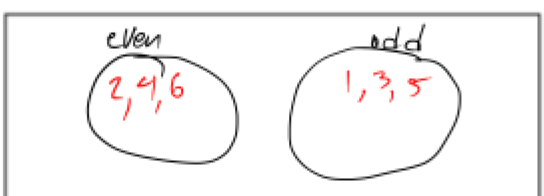
iii. Event A or B (written Event A ∪ B) ← or

{1, 2, 3, 4, 5, 6}

iv. Event A and B (written Event A ∩ B) ← and

{ } empty   {∅} nothing

b) Draw a Venn Diagram for the Sample Space and indicate where each event would go



c) Calculate the following probabilities

even

i. P(A) = $\frac{3 \text{ favorable}}{6 \text{ total}} = \frac{1}{2} = 50\%$

odd

ii. P(B) = $\frac{3 \text{ fav.}}{6 \text{ tot}} = \frac{1}{2} = 50\%$

iii. P(A U B) = $\frac{6 \text{ fav.}}{6 \text{ tot}} = 1 = 100\%$

or

iv. P(A ∩ B) = $\frac{0 \text{ fav}}{6 \text{ tot}} = 0\%$

and

Since P(A ∩ B) = 0, we call Event A and Event B Mutually exclusive.

Another way to see this: Since Event A and Event B do **not** overlap in the Venn Diagram, we call the events Mutually exclusive.

Ex 2

A six-sided die is rolled. Let's call Event A "An even number is rolled" and Event B "A multiple of 3 is rolled"

d) List all the possible outcomes for:

ii. Event A

{2, 4, 6}

ii. Event B

{3, 6}

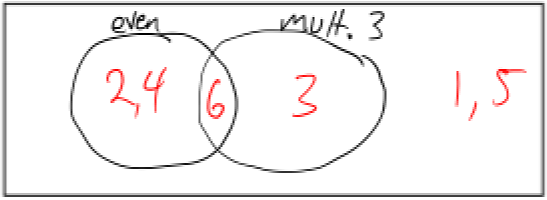
iii. Event A or B (written Event A U B)

{2, 3, 4, 6}

iv. Event A and B (written Event A ∩ B)

{6}

e) Draw a Venn Diagram for the Sample Space and indicate where each event would go



f) Calculate the following probabilities

ii. P(A) = $\frac{3 \text{ fav}}{6 \text{ tot}} = \frac{1}{2} = 50\%$

ii. P(B) = $\frac{2 \text{ fav}}{6 \text{ tot}} = \frac{1}{3} = 33.3\%$

iii. P(A U B) = $\frac{4 \text{ fav}}{6 \text{ tot}} = \frac{2}{3} = 66.7\%$

iv. P(A ∩ B) = $\frac{1 \text{ fav}}{6 \text{ tot}} = \frac{1}{6} = 16.7\%$

and

and

Since P(A ∩ B) ≠ 0, Event A and Event B are **not** mutually exclusive.

Another way to see this: Since Event A and Event B do overlap in the Venn Diagram, we know the events are **not** Mutually exclusive.

The ways to check if two events are mutually exclusive:

1. Think about the probability: P(A ∩ B). Is it possible for Event A and Event B to happen at the same time? If yes, then they are Not mutually exclusive
2. If they don't overlap in a Venn Diagram, then they are mutually exclusive

Ex 3 Are the following events mutually exclusive?

a) You draw a card from a standard deck

Event A - A face card is selected   Event B – A club is selected

J, Q, K

Can they happen at same time?

Yes ⇒ $\boxed{\text{NOT}}$ mutually exclusive

b) You roll two 6-sided dice

Event A – Both dice show the same number   Event B – The dice add to 9

1, 1 = 2
2, 2 = 4
3, 3 = 6
4, 4 = 8
5, 5 = 10
6, 6 = 12

Can they happen at the same time?

NO ⇒ mutually exclusive

Ex 4

For each experiment below, think of two events that are **mutually exclusive** and two events that are **NOT mutually exclusive**.

a) Drawing a card from a standard deck

can both happen at same time

Can't both happen at same time

i. Two mutually exclusive events

face (J, Q, K)	Red
Ace	Club

ii. Two **Not** mutually exclusive events

three	Red
Red	heart

b) Rolling a 20 sided die (D20)

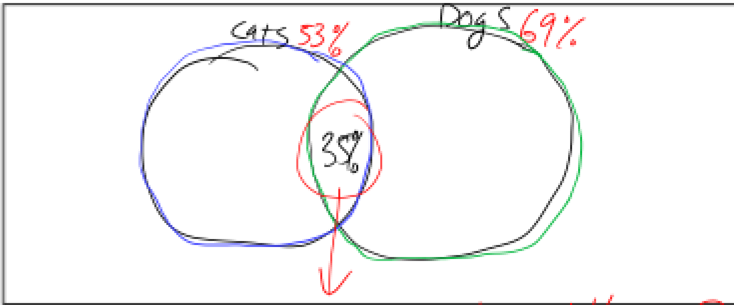
ii. Two mutually exclusive events

even	mult 10 ← 10, 20
one	mult 7 ← 7, 14

ii. Two Not mutually exclusive events

20	odd
even	mult

There is a formula we can use that calculates P(A U B): Let's look back at the warm-up



Double Counted middle ⇒ $P(A \cap B)$

or

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Let's check that the formula works for **Warm-up**, **Ex 1**, and **Ex 2** from before:

Check **Warm-up**

$$P(\text{Cats} \cup \text{Dogs}) = P(\text{Cats}) + P(\text{Dogs}) - P(\text{Cats} \cap \text{Dogs})$$
$$= 53\% + 69\% - 35\% = \boxed{87\%}$$

Check **Ex 1**

$$P(\text{even} \cup \text{odd}) = P(\text{even}) + P(\text{odd}) - P(\text{even} \cap \text{odd})$$
$$= \frac{1}{2} + \frac{1}{2} - 0$$ ← mutually exclusive
$$= 1 = \boxed{100\%}$$

Check **Ex 2**

$$P(\text{even} \cup \text{mult. } 3) = P(\text{even}) + P(\text{mult } 3) - P(\text{even} \cap \text{mult } 3)$$
$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = 0.666\ldots$$
$$= \boxed{66.7\%}$$