number, face cards (J, Q, K) are worth 10 and Aces are worth 1 or 11 (whichever is better for you). So getting an Ace and a ten gets you 21 ("Blackjack")! Find:

a) P(First card is an Ace) $= \frac{4 \text{ fav}}{52 \text{ total}} = 7.7\%$

b) P(First card is a face card) $= \frac{12}{52} = 23.1\%$

c) P(First card is a face card and second card is an ace) $= \left(\frac{12}{52}\right) \cdot \left(\frac{4}{51}\right) = \underline{1.8\%}$

AND

mult

1 card is out

d) Why can't you determine P(Second card is an ace) on its own?

P(Second card is an ace) depends on what the first card is!

1st card Ace $\Rightarrow \frac{3}{51}$

1st card Not Ace $\Rightarrow \frac{4}{51}$

If the probability of Event B depends on whether or not Event A has occurred, Events A and B are called dependent events.

If the probability of Event B DOES NOT depend on whether or not Event A has occurred, Events A and B are called independent events.

Independent Events	Dependent Events
• two dice rolls • two coin flips • draw two cards with replacement	• 1st and 2nd card drawn from a deck • Breaking a bone if you have broken it before

How could we change the rules of Blackjack so that every card drawn is an independent event?

Play with replacement

Let's go back to Blackjack again. Let A = {first card is a face card} B = {second card is an ace}
When we calculated the probability for part c) of the warm-up, we used the fact that:

and and

$$P(A \cap B) = P(A) \times P(B \text{ given that } A \text{ happened})$$

Given that

We use the notation: P(B given that A happened) = $P(B|A)$

So the formula is: $P(A \cap B) = P(A) \overset{\text{and}}{\cdot} P(B|A)$ ← B affected by A

NOTE: $P(A \cap B) = P(B \cap A)$ even though the calculations are different

Ex 1

Two cards are drawn from a deck while playing Blackjack (**without** replacement). Use the formula to determine the probability of the following events:

a) Both cards are face cards

A - 1st face B - 2nd face

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$= \left(\frac{12}{52}\right) \cdot \left(\frac{11}{51}\right)$$

$$= 4.98\%$$

b) One card is a face card and the other is an ace (*different from the warmup!*)

P(Face then Ace) or P(Ace then Face)

$$= \left(\frac{12}{52}\right) \cdot \left(\frac{4}{51}\right) + \left(\frac{4}{52}\right) \cdot \left(\frac{12}{51}\right)$$

$$= 3.62\%$$

Ex 2

Two cards are drawn from a deck **with** replacement. Determine the probability of the following events:

Ex 2

Two cards are drawn from a deck **with** replacement. Determine the probability of the following events:

a) Both cards are face cards

$P(A \cap B) = P(A) \cdot P(B|A)$

$= \frac{12}{52} \cdot \frac{12}{52}$ ← P(B) ← Doesn't care about A anymore

$= 5.32\%$

b) One card is a face card and the other is an ace

P(Face then Ace) or P(Ace then Face)

$\left(\frac{12}{52}\right) \cdot \left(\frac{4}{52}\right)$ or $\left(\frac{4}{52}\right) \cdot \left(\frac{12}{52}\right)$ ← Doesn't care about 1st card

$= 3.55\%$

When the two events, A and B, are independent:

P(B|A) = P(B)

So, with two independent events, the formula changes to:

P(A ∩ B) = P(A) · P(B) ← B is not affected anymore

It's really easy to confuse **mutually exclusive** and **independent/dependent** events.

- **Mutually exclusive** events involve whether two events can happen at the same time
- **Independent/dependent** events involve one event affecting the probability of another event

Are Event A and B…	Mutually Exclusive?	Independent/Dependent?
You pick a random Byng student A: They are over 6 feet tall B: They are over 200 pounds	NO, Both can happen at same time	Dependent, weight/height are related
You draw a card from a deck A: It's a club B: It's a red card	yes, Can't both happen at same time	Dependent, if it's a club, can't be red ⇒ A happening affects the probability of B

Useful formulae so far:

P(A ∪ B) = P(A) + P(B) - P(A ∩ B)　　　P(A ∩ B) = P(A) • P(B|A)

Ex 3

If $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, use appropriate formulas to determine:

a) P(A ∩ B)　　　b) P(B|A)　　　c) P(A|B)