

• 52 cards total

• 26 black cards
(Clubs and Spades)

• 26 red cards
(Hearts and Diamonds)

• 13 cards of each suit
(Clubs, Spades, Hearts, Diamonds)

• 4 Cards of each type
(A, 2-10, J, Q, K)

1. How many 5-card hands can be made with:
- exactly 3 Diamonds?

$${}_{13}C_3 \cdot {}_{39}C_2 = 211926$$

Diamonds Other

- at most 2 red cards?

OR, $50C_5 \oplus 12C_4 \cdot 40C_1 \oplus 2C_2 \cdot 30C_3$

$$= {}_{26}C_5 + {}_{26}C_1 \cdot {}_{26}C_4 + {}_{26}C_2 \cdot {}_{26}C_3$$

$$= 65780 + 368700 + 845000$$

$$= \underline{1299480}$$

- exactly 2 kings and 2 queens?

$$4C_2 \cdot 4C_2 \cdot 44C_1 = 1584$$

king Queen Other

- at least 1 Club?

(Use complement for full marks)

$$\text{total} = 0C_5, 50C_1 \oplus 1C_4 \cdot 40C_1 \oplus 2C_2 \cdot 30C_3$$

$$\oplus 3C_2 \cdot 20C_1 \oplus 4C_1 \cdot 10C_1 \oplus 5C_1$$

x = at least 1 club

$$x = \text{total} - 0C_5, 50C_1$$

$$= {}_{52}C_5 - {}_{39}C_5 = \underline{2023203}$$

2. How many ways can we arrange the letters in the word "WORKBOOK" if:

- no restrictions?

8 letters, 3 O's 2 K's

$$\frac{8!}{2!3!} = 3360$$

- the first two letters are both "O"?

OO _____

6 letters, 2 K's

$$\frac{6!}{2!} = \underline{360}$$

- the letter "K" must be last?

_____ K

7 letters, 3 O's

$$\frac{7!}{3!} = 840$$

- all the "K"s are together?

KK O O O W R B

can't reorder 7 'tiles' to arrange, 3 O's

$$\frac{7!}{3!} = \underline{840}$$

3. Mr. G is coaching the Lord Byng Junior Volleyball team. He has 14 players on the team in total.

- a. If he needs to choose 6 of them (positions don't matter) for the starting lineup, and Tim (team captain) must be on the starting lineup, how many starting lineups are possible?

$$1C_1 \cdot 13C_5 = \underline{1287}$$

Tim others

- b. If he needs to choose 6 of them (positions don't matter) for the starting lineup, but Sam and Howard can't both be on the starting lineup, how many lineups are possible?

Yes H or Yes S or No S
No S No H No H

$$= 1C_1 \cdot 12C_5 + 1C_1 \cdot 12C_5 + 12C_6 = \underline{2508}$$

- c. Mr. G is making a promo poster for the team. He wants 9 total students standing in a row, with Tim in the middle, for a picture on the poster. How many ways can Mr. G arrange the team members for the photo?

Tim

$$\underline{(13) \cdot (12) \cdot (11) \cdot (10) \cdot (1) \cdot (1) \cdot (8) \cdot (7) \cdot (6)} = \underline{51891840}$$

4. A school schedule has 8 blocks (4 blocks day 1; 4 blocks day 2). A student has to choose 5 academic courses and 3 elective courses. There are 9 different academic courses and 12 different elective courses to choose from. If each course is offered every block and schedule order is important, how many different schedules can be made?

① choose courses $\Rightarrow 9C_5 \cdot 12C_3$

AND

② Arrange schedule $\Rightarrow 8$ Courses $\Rightarrow 8!$
to arrange

$$\text{total} = 9C_5 \cdot 12C_3 \cdot 8! = \underline{1117670400}$$